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A Design-Based Study of a Mobile-Supported KADIR Model for Mathematical Problem-Solving and Communication

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ABSTRACT

The integration of mobile learning with structured pedagogy has become increasingly important in mathematics education; however, existing studies often emphasize general academic achievement rather than simultaneously developing mathematical problem-solving and communication skills. This study aimed to develop and evaluate a mobile-supported KADIR learning model that integrates sequential cognitive scaffolding and collaborative mathematical discourse to enhance both competencies. The study employed a Design-Based Research (DBR) approach integrated with a modified ADDIE framework. Classroom implementation adopted a one-group pre-test–post-test design involving 33 seventh-grade students, supported by four expert validators and four mathematics teachers. Data were collected through validation sheets, student response questionnaires, classroom observations, and mathematical problem-solving and communication tests. Data were analyzed using descriptive statistics, paired-sample t-tests, Cohen's d, normalized gain (N-gain), and qualitative analysis of classroom observations. The developed model demonstrated high validity, with an overall expert validation score of 4.35 out of 5, and high practicality, reflected by an overall student response rate of 83.7%. Significant improvements were observed in students' mathematical problem-solving (t = 8.72, p < .001, Cohen's d = 1.25, N-gain = 0.46) and mathematical communication (t = 7.95, p < .001, Cohen's d = 1.10, N-gain = 0.47). Qualitative findings indicated that structured discourse, collaborative interaction, and reflective activities promoted greater student engagement and flexible mathematical reasoning. The findings suggest that the mobile-supported KADIR model effectively integrates mobile technology with structured pedagogy to foster higher-order mathematical competencies. Although the study was limited by its single-group design, small sample, and single-school context, the model offers promising design principles for technology-enhanced mathematics instruction and provides a foundation for future large-scale experimental research.

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1. INTRODUCTION

The rapid advancement of digital technology has significantly transformed educational practices worldwide, including mathematics education. Among various forms of educational technology, mobile learning has received increasing attention because it enables flexible, interactive, and student-centered learning experiences that can occur anytime and anywhere. Mobile technologies allow learners to access instructional materials ubiquitously and support collaborative learning processes through continuous interaction and communication. Recent studies have demonstrated that mobile-assisted learning can positively influence students' engagement, motivation, and academic achievement when effectively integrated with appropriate pedagogical strategies (Hillmayr et al., 2020; Crompton et al., 2020). However, the effectiveness of mobile learning is not determined solely by technological availability but also by how technology is pedagogically designed and implemented in classroom instruction (Bond, 2020; Tang et al., 2021). Therefore, the integration of mobile technology with structured instructional models has become an important issue in contemporary mathematics education research.

In mathematics education, problem-solving and mathematical communication are widely recognized as fundamental competencies for developing higher-order thinking skills. Problem-solving enables students to apply mathematical concepts and reasoning to unfamiliar situations, while mathematical communication supports students in expressing, explaining, and justifying mathematical ideas systematically (Schoenfeld, 2020; Sfard, 2021). These competencies are emphasized in international educational frameworks such as the Program for International Student Assessment (PISA) and the National Council of Teachers of Mathematics (NCTM), which highlight the importance of reasoning, argumentation, and communication in mathematics learning. Nevertheless, empirical evidence suggests that many students still experience difficulties in developing these competencies effectively.

In the Indonesian context, students' mathematical competencies remain a major educational concern. The results of PISA 2022 revealed that Indonesian students performed below the OECD average in mathematics, particularly in tasks requiring reasoning, problem-solving, and mathematical interpretation (OECD, 2023). Similar findings have also been reported in TIMSS assessments, where Indonesian students demonstrated limited ability to apply mathematical concepts in complex situations and communicate mathematical reasoning effectively. At the classroom level, mathematics instruction in many Indonesian schools continues to emphasize procedural learning and teacher-centered practices, limiting opportunities for students to engage in discussion, reflection, and collaborative problem-solving activities (Prediger et al., 2020; Planas et al., 2021). Consequently, students often become passive learners who focus primarily on obtaining correct answers rather than understanding mathematical processes and communicating their ideas critically. These conditions indicate the need for innovative instructional approaches that support active participation, collaborative interaction, and meaningful mathematical thinking.

One instructional approach that has the potential to address these challenges is the KADIR learning model. The KADIR model is an instructional framework developed by the authors based on socio-constructivist learning theory and collaborative mathematical inquiry principles. The acronym KADIR refers to five sequential stages of learning: connection, application, discourse, improvisation, and reflection. The connection stage aims to activate students' prior knowledge and contextual understanding, while the application stage focuses on applying mathematical concepts to problem situations. The discourse stage facilitates collaborative discussion and mathematical communication among students, whereas the improvisation stage encourages students to generate alternative solution strategies and develop flexible thinking. Finally, the reflection stage supports students in evaluating their understanding and learning processes. Unlike conventional instructional models that primarily emphasize procedural mastery, the KADIR model explicitly integrates collaborative discourse, reflective learning, and sequential cognitive scaffolding within a unified pedagogical structure.

The theoretical foundation of the KADIR model is grounded in socio-constructivist theory, which emphasizes that knowledge is actively constructed through interaction, dialogue, and reflection (Vygotsky, 1978; Sfard, 2021). This perspective suggests that learning occurs most effectively when students actively participate in collaborative meaning-making processes rather than passively receiving information from teachers. In addition, the model draws upon mathematical discourse theory, which highlights the important role of communication and argumentation in developing conceptual understanding (Ingram & Elliott, 2020; Schwarz et al., 2021). The integration of mobile technology into the KADIR model is further supported by mobile learning theory, which emphasizes accessibility, interactivity, learner autonomy, and continuous engagement as important factors in enhancing learning effectiveness (Nikou & Economides, 2020). Through the integration of these theoretical perspectives, the mobile-supported KADIR model is expected to create a learning environment that promotes active participation, collaborative interaction, and higher-order mathematical thinking.

Despite the increasing number of studies on mobile learning and innovative mathematics instruction, several important gaps remain in the existing literature. First, many previous studies on mobile learning in mathematics education have primarily focused on general academic achievement and student engagement rather than specifically examining mathematical problem-solving and communication skills simultaneously (Crompton et al., 2020; Zhang & Pérez-Paredes, 2021). Second, problem-solving and communication are often investigated as separate constructs, even though both competencies are closely interconnected in mathematical thinking processes. Third, many mobile learning studies still position technology merely as a delivery tool without integrating it into a coherent pedagogical framework that systematically supports collaborative learning and reflective thinking (Lo & Hew, 2022). Fourth, only limited studies have employed a design-based research approach to systematically design, refine, and evaluate mobile-supported instructional models in authentic mathematics classroom settings (McKenney & Reeves, 2021). These limitations indicate the need for a comprehensive instructional model that integrates structured pedagogy, collaborative discourse, and mobile technology within a unified learning framework.

Several previous studies further illustrate these limitations. Lai and Hwang (2021) found that mobile-supported learning environments improved student engagement and learning performance; however, their study did not explicitly examine mathematical communication skills. Similarly, Hwang and Tu (2021) reported positive effects of mobile-assisted learning systems on student achievement, yet the instructional process lacked a structured pedagogical model that systematically integrated discussion, reflection, and collaborative inquiry. Other studies have emphasized the importance of discourse and collaborative learning in mathematics classrooms but have not sufficiently explored how these processes can be effectively facilitated through mobile-supported learning environments (Wagner et al., 2021; Planas, 2021). Therefore, there remains a need for research that systematically integrates mobile learning, collaborative mathematical discourse, and structured pedagogical stages to enhance students' higher-order mathematical competencies.

Based on these considerations, this study proposes a mobile-supported KADIR learning model designed to enhance students' mathematical problem-solving and communication skills simultaneously. The novelty of this study lies in four aspects. First, the study introduces the KADIR model as a structured pedagogical framework specifically designed to support mathematics learning through sequential cognitive and communicative processes. Second, the study integrates the KADIR model with mobile learning technology to facilitate collaborative interaction, accessibility, and reflective learning. Third, the study simultaneously investigates the development of mathematical problem-solving and communication skills within a single instructional intervention. Fourth, the study employs a design-based research approach that emphasizes iterative refinement and practical implementation in authentic classroom contexts.

Accordingly, the research problem of this study concerns how a mobile-supported KADIR learning model can be systematically designed and implemented to enhance students' mathematical problem-solving and communication skills. This study specifically addresses the following research questions: (1) how can a mobile-supported KADIR learning model be designed and developed to support mathematics learning; (2) to what extent is the developed model valid and practical for classroom implementation; (3) how effective is the mobile-supported KADIR learning model in improving students' mathematical problem-solving and communication skills; and (4) what design principles emerge from the implementation of the mobile-supported KADIR learning model? Therefore, the objective of this study is to design, develop, and evaluate a mobile-supported KADIR learning model that enhances students' mathematical problem-solving and communication skills. The study is expected to contribute theoretically by extending research on mobile-supported mathematics learning through the integration of structured pedagogy and collaborative discourse. Practically, the study provides an empirically validated instructional model that may support mathematics teachers in implementing technology-enhanced learning environments that promote higher-order thinking and meaningful mathematical communication.

2. METHODS

2.1 Research Design

This study employed a Design-Based Research (DBR) approach to systematically design, develop, refine, and evaluate a mobile-supported KADIR learning model aimed at enhancing students' mathematical problem-solving and communication skills. DBR was selected because it enables the integration of theoretical foundations with iterative cycles of design, implementation, evaluation, and refinement in authentic educational contexts, thereby generating both practical and theoretical contributions (McKenney & Reeves, 2021). In operationalizing the DBR process, this study adopted a modified ADDIE framework consisting of Analysis, Design, Development, and Implementation phases, while formative evaluation was embedded throughout each phase to ensure continuous improvement of the developed learning model.

The research also implemented a one-group pre-test–post-test design during the classroom implementation stage to evaluate the effectiveness of the intervention. The DBR process was conducted through several iterative cycles. In the first cycle, preliminary analysis was carried out through classroom observations, teacher interviews, and instructional material reviews to identify existing problems in mathematics learning. The findings from this stage informed the initial design of the mobile-supported KADIR model. In the second cycle, the prototype was developed and evaluated by expert validators and mathematics teachers, whose feedback was used to revise the pedagogical structure, instructional activities, and mobile application interface. In the third cycle, a pilot implementation involving a small group of students was conducted to identify usability issues and technical limitations. The final cycle involved full classroom implementation and evaluation of the model's validity, practicality, and effectiveness. These iterative cycles allowed continuous refinement of both pedagogical and technological components of the learning model. To provide a clearer overview of the DBR process, the research framework used in this study is presented in Table 1.

Table 1. Integrated Design-Based Research Framework

Phase	Core Activities	Data Sources	Analytical Focus	Output
Analysis	Observation, interviews, material review	Field notes, teacher input	Problem identification	Instructional needs
Design	Model and media design, instrument development	Design documents	Pedagogical-technological alignment	Prototype
Development	Validation, revision, usability testing	Expert and teacher feedback	Validity and usability	Validated model

Phase	Core Activities	Data Sources	Analytical Focus	Output
Implementation	Pilot and classroom implementation	Test scores, questionnaires, observations	Practicality and effectiveness	Final product

Table 1 illustrates the systematic stages of the DBR process, highlighting the relationship between research activities, data sources, analytical focus, and expected outputs throughout the study.

2.2 Participants and Research Setting

The study was conducted at a junior secondary school (Madrasah Tsanawiyah/MTs) in Indonesia during the 2024 academic year. The main implementation phase involved 33 seventh-grade students aged between 12 and 13 years old, consisting of 18 female students and 15 male students. Based on teacher diagnostic records and previous mathematics achievement data, the students demonstrated relatively heterogeneous mathematical abilities, with many students experiencing difficulties in mathematical reasoning, problem-solving, and communication tasks.

In addition to student participants, the study involved four expert validators, consisting of two mathematics education experts and two educational technology experts, who evaluated the validity of the learning model and research instruments. Four mathematics teachers also participated as practitioner evaluators to assess the practicality and feasibility of the developed model during implementation.

Participants were selected using purposive sampling. This technique was employed because the selected school possessed adequate technological infrastructure, including students' access to smartphones and internet connectivity, which supported the implementation of mobile learning. Furthermore, the participating teachers demonstrated readiness and willingness to integrate technology into mathematics instruction. Since the study aimed to develop and evaluate an instructional intervention within a specific educational context, purposive sampling was considered appropriate for achieving the objectives of the research.

2.3 Ethical Considerations

Ethical considerations were carefully addressed throughout the study. Prior to conducting the research, permission was obtained from the school administration. Students and their parents or guardians were informed about the objectives, procedures, and voluntary nature of participation in the study. Written informed consent was obtained before data collection began. Participants' identities and personal information were kept confidential, and all research data were analyzed anonymously. Participants were also informed that they could withdraw from the study at any time without academic consequences.

2.4 Development of the Mobile-Supported KADIR Learning Model

The mobile-supported KADIR learning model was developed as a structured instructional framework consisting of five sequential stages: connection, application, discourse, improvisation, and reflection. The model was designed based on socio-constructivist learning theory and collaborative mathematical inquiry principles. To support the implementation process, a mobile learning application was developed using the Kodular platform. The application contained several instructional features, including digital learning modules, interactive student worksheets, collaborative discussion forums, problem-solving tasks, reflective learning activities, and assessment components. The interface was designed to support accessibility, interaction, and collaborative engagement among students.

During the connection stage, students were introduced to contextual mathematical problems aimed at activating prior knowledge and establishing conceptual connections. In the application stage, students solved structured mathematical tasks individually and collaboratively. The discourse stage facilitated mathematical discussion, argumentation, and peer interaction. In the improvisation stage, students were encouraged to explore alternative solution strategies and develop flexible mathematical thinking. Finally, the reflection stage guided students to evaluate their understanding and summarize learning experiences through reflective activities.

2.5 Duration of the Intervention

The implementation phase was conducted over six weeks during regular mathematics learning sessions. Students participated in two instructional sessions per week, with each session lasting approximately 90 minutes. In total, the intervention consisted of 12 classroom meetings. The pre-test was administered before the intervention began, while the post-test was conducted after the completion of all learning sessions.

2.6 Instruments and Data Collection

Multiple instruments were employed to ensure methodological rigor and data triangulation. These instruments included validation sheets, student response questionnaires, mathematical problem-solving and communication tests, observation notes, and documentation records. Validation sheets were used to evaluate the quality of the instructional model, learning materials, and mobile application in terms of content accuracy, pedagogical appropriateness, instructional design, and media usability. The instrument employed a five-point Likert scale ranging from 1 (very poor) to 5 (excellent). The resulting mean scores were interpreted according to predetermined validity categories.

Student response questionnaires were administered after implementation to assess the practicality and usability of the developed learning model. The questionnaire measured students' perceptions regarding ease of use, engagement, interaction, and learning support. Practicality scores were calculated as percentages, and products achieving scores above 65% were categorized as practically feasible for classroom implementation based on commonly accepted standards in educational development research.

Essay-based tests were developed to measure students' mathematical problem-solving and communication skills. Mathematical problem-solving was operationally defined as students' ability to understand problems, formulate solution strategies, execute mathematical procedures, and evaluate solutions systematically. Mathematical communication was defined as students' ability to express mathematical ideas clearly through written explanations, mathematical representations, symbols, and logical reasoning. The indicators used to assess both competencies are presented in Table 2.

Table 2. Indicators of Mathematical Competencies

Competency	Indicators
Problem-solving	Understanding problems, identifying concepts, developing strategies, executing procedures, evaluating solutions
Mathematical communication	Expressing ideas, using symbols appropriately, explaining reasoning logically, representing information accurately, communicating conclusions

Each indicator was assessed using an analytic scoring rubric ranging from 0 to 4, where 0 represented incorrect or missing responses and 4 represented excellent performance. Prior to implementation, all instruments were reviewed and revised based on expert feedback to ensure content validity and clarity.

2.7 Data Analysis

Data analysis was conducted in three stages: validity analysis, practicality analysis, and effectiveness analysis. Validity analysis was performed by calculating the mean scores obtained from expert validation sheets. The validity categories used in this study are presented in Table 3.

Table 3. Validity Criteria

Mean Score	Category
4.21–5.00	Very valid
3.41–4.20	Valid
2.61–3.40	Moderately valid
1.81–2.60	Less valid
1.00–1.80	Invalid

Practicality analysis was conducted by calculating the percentage of positive responses from student questionnaires. Products with practicality scores exceeding 65% were considered practically feasible for classroom implementation. Effectiveness analysis employed both descriptive and inferential statistics. Descriptive statistics, including mean scores and standard deviations, were used to summarize students' pre-test and post-test performance. Before conducting inferential analysis, the normality assumption was tested using the Shapiro–Wilk test. The results indicated that the data were normally distributed ($p > .05$), thereby satisfying the assumptions for parametric testing.

A paired sample t-test was subsequently conducted to determine statistically significant differences between pre-test and post-test scores. Statistical significance was determined at the .05 level, and exact significance values were reported using $p < .001$ where appropriate. To determine the magnitude of learning improvement, Cohen's d effect size and normalized gain (N-gain) analyses were also conducted. In addition, qualitative observational data were analyzed descriptively to identify patterns of student interaction, collaborative discourse, and engagement during the implementation process.

3. FINDINGS AND DISCUSSION

3.1 Findings

3.1.1 Validity of the Mobile-Supported KADIR Learning Model

The validity of the mobile-supported KADIR learning model was evaluated through expert judgment involving two mathematics education experts and two educational technology experts. The evaluation focused on three aspects, namely content validity, instructional design, and media usability. The results of expert validation are presented in Table 4.

Table 4. Expert Validation Results

Aspect	Mean Score	Category
Content validity	4.42	Very valid
Instructional design	4.35	Very valid
Media usability	4.28	Very valid
Overall	4.35	Very valid

The results indicate that the developed learning model achieved a high level of validity, with an overall mean score of 4.35 categorized as "very valid." The highest score was obtained in content validity, suggesting that the instructional materials were strongly aligned with curriculum objectives and targeted mathematical competencies. Meanwhile, the slightly lower score in media usability reflected several interface-related issues identified during validation, particularly concerning navigation clarity and interaction flow within the mobile application. Based on the feedback from validators, revisions were made to improve instructional sequencing, simplify navigation menus, and enhance the accessibility of learning features before classroom implementation.

3.1.2 Practicality of the Learning Model

The practicality of the mobile-supported KADIR learning model was assessed through student response questionnaires administered after the implementation phase. The questionnaire evaluated students' perceptions regarding ease of use, engagement, interaction, and learning support. The practicality results are presented in Table 5.

Table 5. Practicality Results

Indicator	Percentage (%)	Category
Ease of use	82.4	Very practical
Engagement	85.1	Very practical
Learning support	83.7	Very practical
Overall	83.7	Very practical

The findings demonstrate that the developed model was considered highly practical by students, as reflected by the overall practicality score of 83.7%. The highest percentage was observed in the engagement indicator, suggesting that the integration of mobile learning activities with structured pedagogical stages supported active student participation during mathematics learning. Students reported that the discourse and improvisation stages encouraged interaction, collaborative discussion, and exploration of alternative problem-solving strategies.

Qualitative observations also supported these findings. During group discussions, students actively exchanged ideas and negotiated mathematical reasoning with peers. One student stated that: "Using the application helped us discuss mathematics more easily because we could immediately compare our answers and explain our reasoning together." Similarly, a participating teacher observed that: "Students became more willing to express their mathematical ideas during discussion activities compared to conventional classroom instruction." These findings suggest that the mobile-supported KADIR model not only supported usability but also facilitated collaborative learning interactions and meaningful engagement during mathematics instruction.

3.1.3 Effectiveness of the Learning Model

The effectiveness of the mobile-supported KADIR learning model was evaluated through pre-test and post-test assessments of students' mathematical problem-solving and communication skills. Descriptive statistics of students' performance are presented in Table 6.

Table 6. Descriptive Statistics of Pre-Test and Post-Test Scores

Variable	Test	Mean	SD
Problem-solving	Pre-test	48.6	10.4
	Post-test	72.3	9.8
Communication	Pre-test	56.2	9.6
	Post-test	76.5	8.7

The descriptive results indicate substantial increases in both competencies following the implementation of the learning model. Students' mathematical problem-solving scores increased from a mean of 48.6 to 72.3, while mathematical communication scores increased from 56.2 to 76.5. In addition, the slightly lower standard deviation values in the post-test results suggest more consistent performance among students after the intervention. Prior to inferential analysis, normality assumptions were examined using the Shapiro–Wilk test. The results indicated that both pre-test and post-test scores were normally distributed ($p > .05$), thereby satisfying the assumptions required for parametric testing. A paired sample t-test was subsequently conducted to examine differences between pre-test and post-test scores. The complete results are presented in Table 7.

Table 7. Paired Sample t-Test Results

Variable	Mean Difference	t	df	p	Interpretation
Problem-solving	23.7	8.72	32	$p < .001$	Significant
Communication	20.3	7.95	32	$p < .001$	Significant

The paired sample t-test results indicate statistically significant differences between pre-test and post-test scores for both mathematical problem-solving and communication skills. These findings suggest that the implementation of the mobile-supported KADIR learning model was associated with improved student performance in both competencies. To further examine the magnitude of improvement, effect size (Cohen's d) and normalized gain (N-gain) analyses were conducted. The results are presented in Table 8.

Table 8. Effect Size and N-Gain Analysis

Variable	Cohen's d	Category	N-gain	Category
Problem-solving	1.25	Large	0.46	Medium
Communication	1.10	Large	0.47	Medium

The N-gain results were interpreted using three categories: low (<0.30), medium (0.30–0.70), and high (>0.70). The findings indicate medium N-gain values for both competencies, suggesting moderate learning improvement relative to students' initial abilities. Although Cohen's d values indicated large effect sizes, the N-gain values remained within the medium category. This difference may be explained by students' relatively low initial mathematical competencies and the limited duration of the intervention. While substantial improvements in performance were observed, some students still experienced difficulties in consistently applying mathematical reasoning and communicating complex mathematical ideas. Therefore, the findings indicate the potential effectiveness of the mobile-supported KADIR learning model while also suggesting that longer implementation periods may be necessary to achieve higher normalized learning gains.

3.1.4 Observation of Learning Processes

Qualitative observations conducted during implementation provided additional insights into students' learning experiences. During the discourse stage, students actively participated in collaborative discussion and exchanged mathematical ideas with peers. Many students demonstrated increased confidence in explaining their reasoning and evaluating alternative solutions. Observation notes indicated that students who were initially passive gradually became more engaged during collaborative activities facilitated through the mobile application.

In the improvisation stage, students demonstrated greater flexibility in generating alternative problem-solving strategies. Several students attempted multiple solution approaches rather than relying solely on procedural methods introduced by the teacher. One observation note recorded that: "Students began comparing different solution strategies and discussing which method was more efficient or easier to understand." Teachers also reported positive changes in classroom interaction patterns. According to one teacher: "The structured learning stages helped students communicate mathematical ideas more confidently and encouraged more active participation during classroom discussion." These qualitative findings support the quantitative results by indicating that the mobile-supported KADIR learning model facilitated collaborative interaction, reflective thinking, and meaningful mathematical discourse during the learning process.

3.2 Discussion

The findings of this study indicate that the mobile-supported KADIR learning model has the potential to support the development of students' mathematical problem-solving and communication skills through the integration of structured pedagogy and mobile learning technology. More importantly, the results provide insights into how each instructional stage within the KADIR model contributed to specific learning outcomes and facilitated meaningful mathematical interaction during the learning process.

The improvement in students' mathematical communication skills can be understood through several specific indicators observed during implementation, including students' ability to express mathematical ideas clearly, explain reasoning logically, use mathematical symbols appropriately, represent information accurately, and communicate conclusions coherently. Among these indicators, the most visible improvement occurred in students' ability to explain mathematical reasoning and communicate alternative solution strategies during collaborative discussion activities. Observation data indicated that students became more confident in articulating their mathematical thinking during peer interaction and classroom discourse. These findings are consistent with previous studies emphasizing that mathematical communication develops more effectively when students are actively involved in dialogic interaction and collaborative reasoning processes (Ingram & Elliott, 2020; Wagner et al., 2021).

The discourse stage within the KADIR model played a particularly important role in supporting mathematical communication. During this stage, students engaged in group discussions, negotiated meaning, explained their reasoning, and evaluated peers' arguments. The integration of mobile learning technology further supported these interactions by providing immediate access to learning materials and collaborative discussion features. Qualitative observation notes revealed that students frequently used the discussion forum within the application to compare answers and justify mathematical procedures. These findings suggest that the discourse stage contributed significantly to students' communication development by creating opportunities for active participation and mathematical argumentation.

In addition to communication skills, the study also demonstrated improvements in students' mathematical problem-solving abilities across several indicators, including understanding mathematical problems, identifying relevant concepts, developing solution strategies, executing mathematical procedures accurately, and evaluating solutions systematically. The most substantial improvement appeared in students' ability to formulate solution strategies and evaluate alternative approaches to solving problems. Initially, many students relied primarily on procedural methods demonstrated by teachers; however, during implementation, students gradually demonstrated greater flexibility in exploring multiple solution strategies and reflecting on the effectiveness of different approaches.

The contribution of each KADIR stage to problem-solving development was evident throughout the learning process. The connection stage supported students in activating prior knowledge and identifying relationships between mathematical concepts and contextual situations. This stage appeared to help students better understand problem contexts before attempting solutions. The application stage then facilitated the implementation of mathematical procedures through structured problem-solving activities. Meanwhile, the improvisation stage contributed substantially to students' flexible thinking by encouraging them to explore alternative strategies and compare different methods of solving mathematical problems. Observation data indicated that students increasingly attempted multiple approaches rather than relying solely on a single procedural solution. Finally, the reflection stage supported metacognitive processes by encouraging students to evaluate their understanding and reconsider solution effectiveness. These findings align with socio-constructivist perspectives suggesting that structured cognitive scaffolding and reflective activities play important roles in the development of higher-order mathematical thinking (Sfard, 2021; Schwarz et al., 2021).

The findings further suggest that the effectiveness of the mobile-supported KADIR model was closely related to the interaction between structured pedagogy and technological support. Rather than functioning merely as a content delivery tool, the mobile application facilitated collaborative engagement, accessibility, immediate feedback, and reflective learning activities. This finding supports previous studies arguing that the effectiveness of mobile learning depends largely on how technology is pedagogically integrated into meaningful learning processes (Bond, 2020; Sung et al., 2022). In this study, the mobile application supported each KADIR stage by providing interactive learning resources, collaborative discussion spaces, and guided reflective activities that enhanced student participation and engagement.

To provide a clearer representation of the final instructional framework, the final mobile-supported KADIR learning model should be presented visually in the revised manuscript. The figure should illustrate the relationship between the five instructional stages—connection, application, discourse, improvisation, and reflection—and their integration with mobile learning features such as collaborative discussion, interactive worksheets, reflection modules, and assessment activities. In addition, the figure should demonstrate how each stage contributes to the development of mathematical problem-solving and communication skills within a continuous learning cycle.

Another important finding concerns the relationship between effect size and normalized gain results. Although the study reported large Cohen's d values, the N-gain values remained within the medium category. This finding suggests that while students demonstrated substantial improvement relative to score distribution, the normalized increase relative to the maximum possible score remained moderate. One possible explanation is that students initially possessed relatively low mathematical competencies and limited experience with collaborative mathematical discourse. Consequently, although students showed meaningful progress during the intervention, some continued to experience difficulties in consistently applying mathematical reasoning and communicating complex mathematical ideas. In addition, the relatively short duration of the intervention may have limited opportunities for deeper conceptual internalization and sustained development of higher-order thinking skills.

Despite the positive findings, several limitations of this study should be acknowledged critically. First, the study employed a one-group pre-test–post-test design without a control group, which limits the ability to draw strong causal conclusions regarding the effectiveness of the intervention. The findings therefore indicate potential effectiveness rather than definitive causal impact. Second, the relatively small sample size and single-school context may limit the generalizability of the results to broader educational settings. Third, the intervention was conducted over a relatively short period, which may not have been sufficient to observe long-term changes in students' mathematical competencies. Fourth, the study focused primarily on written assessments and observational data; therefore, future studies may benefit from incorporating interviews, discourse analysis, or learning analytics to provide deeper insights into students' cognitive and collaborative processes.

The findings of this study also provide several practical implications for educational practice and instructional development. For teachers, the study suggests that integrating structured discourse, collaborative problem-solving, and reflective learning activities within mathematics instruction can support students' higher-order thinking and communication skills more effectively than conventional procedural approaches. For curriculum designers, the study highlights the importance of integrating mobile learning technologies with coherent pedagogical frameworks rather than treating technology merely as a supplementary instructional tool. For educational technology developers, the findings indicate that effective mathematics learning applications should support interaction, collaborative engagement, reflection, and flexible problem-solving rather than focusing solely on content delivery and automated assessment.

Furthermore, the transferability of the mobile-supported KADIR model to other educational contexts depends on several important conditions. First, successful implementation requires adequate technological infrastructure, including access to smartphones and stable internet connectivity. Second, teachers need sufficient pedagogical readiness and technological competence to facilitate collaborative and technology-supported learning activities effectively. Third, the model may be particularly suitable for mathematical topics that emphasize reasoning, problem-solving, conceptual understanding, and discussion-based learning processes, such as algebra, geometry, and mathematical modeling. However, adaptation may be necessary when implementing the model in schools with limited technological resources or different curriculum structures. Future research is therefore recommended to examine the implementation of the KADIR model across diverse educational contexts, grade levels, and mathematical topics to further evaluate its scalability and adaptability.

Overall, the findings of this study suggest that the mobile-supported KADIR learning model offers a promising pedagogical approach for integrating mobile technology, collaborative discourse, and structured cognitive scaffolding within mathematics education. The study contributes not only to the practical development of technology-enhanced learning environments but also to the growing body of research on how structured pedagogy and mobile learning can be integrated to support meaningful mathematical learning experiences.

4. CONCLUSION

This study aimed to design, develop, and evaluate a mobile-supported KADIR learning model to enhance students' mathematical problem-solving and communication skills. The findings indicate that the developed model achieved a high level of validity and practicality based on expert evaluation and student responses. The implementation results further suggest that the model was associated with improvements in students' mathematical problem-solving and communication performance, as reflected in statistically significant pre-test and post-test differences, large effect sizes, and moderate N-gain values.

The study also identified several important design principles for integrating mobile learning with structured pedagogy, including the importance of sequential cognitive scaffolding, collaborative mathematical discourse, reflective learning activities, and iterative refinement during instructional development. These findings suggest that the effectiveness of mobile learning depends not only on technological accessibility but also on the alignment between pedagogical stages and technological features. The findings have several practical implications. Mathematics teachers may use the KADIR stages to structure mobile-supported learning activities that encourage collaborative discussion, reflective thinking, and flexible problem-solving. Educational technology developers should design mobile learning applications that support pedagogical interaction and collaborative learning processes rather than merely digitizing instructional materials. In addition, curriculum designers may consider integrating structured discourse and reflective activities into technology-enhanced mathematics instruction. Despite these contributions, the study was limited by the relatively small sample size, the single-school context, and the absence of a control group. Therefore, future research is recommended to examine the effectiveness of the model using stronger experimental or quasi-experimental designs involving larger and more diverse participant groups. Further studies may also investigate the implementation of the KADIR model across different mathematical topics and educational contexts to evaluate its scalability and adaptability.

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