

Metacognitive Processes in Solving Direct and Inverse Proportion Problems: A Qualitative Study of Junior High School Students

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ABSTRACT

Metacognitive regulation plays a crucial role in mathematical problem solving, yet limited qualitative research has examined how students employ planning, monitoring, and evaluation when solving direct and inverse proportion problems. These concepts require students to interpret proportional relationships rather than merely apply computational procedures, making them highly dependent on reflective thinking. This study aimed to analyze the metacognitive processes demonstrated by students with different levels of metacognitive ability while solving proportional reasoning problems. This qualitative descriptive case study involved three Grade VIII students from UPT SMP Negeri 1 Bittuang, representing high, medium, and low metacognitive ability levels. Participants were selected through purposive sampling based on the results of a Metacognitive Awareness Inventory and a proportional reasoning test. Data were collected using problem-solving tasks and semi-structured interviews and analyzed through thematic analysis involving open, axial, and selective coding, supported by triangulation across written work, interview transcripts, and observations. The findings revealed distinct patterns of metacognitive regulation across the three participants. High-metacognitive students demonstrated systematic planning, continuous monitoring, and comprehensive evaluation. Medium-metacognitive students performed adequate planning and procedural monitoring but showed limited conceptual monitoring and evaluation. Low-metacognitive students relied primarily on trial-and-error strategies with minimal evidence of planning, monitoring, or reflective evaluation. Across all ability levels, evaluation emerged as the weakest metacognitive component. The study concludes that metacognitive components develop unevenly and operate as an integrated regulatory system that substantially influences mathematical problem solving. The findings highlight the importance of explicitly incorporating metacognitive scaffolding into mathematics instruction to strengthen students' reflective thinking and proportional reasoning.

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1. INTRODUCTION

Mathematics plays a fundamental role in developing students' logical, critical, and systematic thinking skills. As a core subject in formal education, mathematics not only equips students with computational abilities but also fosters reasoning, problem solving, and decision making skills that are essential in both academic and real-life contexts. However, empirical evidence indicates that Indonesian students' mathematical problem-solving abilities remain relatively low (Sumaryanta et al., 2023), particularly when dealing with contextual and non-routine problems such as direct and inverse proportions. These types of problems require students to go beyond procedural fluency and engage in deeper conceptual understanding, as well as structured and reflective thinking processes. In practice, many students tend to rely on memorized formulas and procedural approaches without fully understanding the problem context, which often leads to ineffective or incorrect solutions (Wijaya et al., 2019). This tendency suggests that students may possess basic computational skills but lack the higher-order thinking processes necessary for meaningful problem solving.

One critical factor influencing students' success in mathematical problem solving is metacognition. Metacognition refers to an individual's awareness and regulation of their own thinking processes, including the ability to plan, monitor, and evaluate cognitive activities (Flavell, 1976). In the context of mathematics learning, metacognitive processes enable students to identify relevant information, select appropriate strategies, monitor their progress during problem solving, and evaluate the correctness and reasonableness of their solutions. These processes are essential for ensuring that students do not merely follow procedures mechanically, but instead engage in thoughtful and strategic problem solving.

Previous studies have consistently shown that students with higher metacognitive ability demonstrate better problem-solving performance, more effective use of strategies, and more accurate self-evaluation (Efklides, 2020; Aditomo & Klieme, 2020; Susanti et al., 2023). Moreover, metacognition has been identified as a key factor in helping students adapt their thinking when encountering unfamiliar or complex problems, making it particularly relevant in topics that require conceptual reasoning such as proportional relationships. Despite extensive research on metacognition in mathematics education, most studies have focused on general problem-solving contexts or have employed quantitative approaches to measure levels of metacognitive awareness. While these studies provide valuable insights, they often do not capture the complexity and depth of students' thinking processes during actual problem solving. In particular, limited attention has been given to in-depth qualitative analyses that explore how students actively engage in metacognitive processes in specific mathematical topics. This gap is especially evident in the context of direct and inverse proportions, which require students to interpret relationships between quantities, distinguish between different types of proportional reasoning, and apply appropriate strategies based on contextual understanding.

Furthermore, few studies explicitly examine how metacognitive processes differ across students with varying ability levels within such specific contexts. As a result, there remains a lack of detailed understanding of how students plan, monitor, and evaluate their thinking when solving proportion problems. Addressing this gap is important because proportional reasoning is a fundamental component of mathematical literacy and is widely applied in everyday situations, such as interpreting rates, comparing quantities, and solving real-world problems. Without strong metacognitive skills, students may struggle to make sense of these relationships and may rely on trial-and-error approaches rather than systematic reasoning. Supporting this concern, preliminary interviews with a Grade VIII mathematics teacher at UPT SMP Negeri 1 Bittuang revealed that students often have difficulty interpreting proportional relationships, tend to depend on peers to identify key information, and rarely engage in monitoring or evaluating their problem-solving processes. These observations indicate that students' difficulties are not only conceptual but also metacognitive in nature, highlighting the need for a deeper and more focused investigation.

Therefore, this study aims to analyze students' metacognitive processes in solving direct and inverse proportion problems by focusing on three key aspects: planning, monitoring, and evaluation. Specifically, this study explores how these processes are manifested across students with high, medium,

and low levels of metacognitive ability. By employing a qualitative approach, this research seeks to provide a more detailed and nuanced understanding of students' thinking processes, which cannot be fully captured through quantitative measures alone. The findings are expected to contribute to the theoretical development of metacognition in mathematics education by providing context-specific insights, as well as to offer practical implications for designing instructional strategies that support students in developing more effective and reflective problem-solving skills.

2. METHODS

This study employed a qualitative approach with a descriptive case study design to obtain an in-depth understanding of students' metacognitive processes in solving direct and inverse proportion problems. The research was conducted at UPT SMP Negeri 1 Bittuang, Lembang Le'tek, Bittuang District, Indonesia. Formal research permission was granted by the institutional review board of the education authority and the headmaster of UPT SMP Negeri 1 Bittuang.

Prior to data collection, informed consent was obtained from the mathematics teacher, the students, and their parents/legal guardians. Participants were informed that their participation was completely voluntary, their identities would remain anonymous (using pseudonyms S1, S2, and S3), and all gathered data would be used strictly for academic purposes. The participants were Grade VIII students selected through purposive sampling. To categorize students' metacognitive ability levels, an initial Metacognitive Awareness Inventory (MAI) adapted for mathematics and a baseline proportional reasoning test were administered to 28 students. The grouping criteria used to select the three primary subjects representing High, Medium, and Low levels are detailed in Table 1.

Table 1. Grouping Criteria for Subject Selection

Category	Score Range	Student Code	Characteristics Observed
High	score > 80	S1	Demonstrates strategic direction, clear goals, consistent checking, and logical validation.
Medium	0 < score < 80	S2	Understands core tasks, applies procedures correctly, but rarely verifies or re-checks final answers.
Low	score < 60	S3	Relies on intuitive trial-and-error, shows mechanical formula-matching, and lacks clear steps.

The primary instruments consisted of a Proportional Reasoning Test (PRT) and semi-structured interview protocols. The instruments were validated by two experts in mathematics education to ensure contextual appropriateness and alignment with the metacognitive framework:

- Task 1 (Direct Proportion): "An active community group in Bittuang can weave 15 traditional Toraja baskets (*Kandian*) in 6 days. If they receive an order of 40 baskets for a local cultural ceremony, how many days will they need to complete the order?"
- Task 2 (Inverse Proportion): "A road renovation project in Lembang Le'tek is planned to be completed in 12 days by 6 construction workers. If the project must be rushed and completed in only 8 days, how many additional workers need to be hired?"

To maintain analysis consistency, students' problem-solving behaviors and interview responses were evaluated against a structured rubric across three metacognitive components table 2.

Table 2. Metacognitive Process Assessment Rubric

Component	Indicators	Score 2 (Full)	Score 1 (Partial)	Score 0 (None)
Planning	Identifying knowns/unknowns; choosing strategies.	Identifies all variables and explicitly states a correct plan.	Identifies some variables but the plan is vague.	Writes random numbers without an explicit plan.
Monitoring	Tracking steps; detecting errors mid-process.	Pauses to check step consistency; corrects self-mid-way.	Proceeds line-by-line but ignores procedural flaws.	Blindly calculates without tracking logic.

Evaluation	Verifying results; contextual validation.	Plugs answer back to test logic; assesses real-world sense.	Compares answer with peers or stops exactly at calculation.	Finishes calculation and turns in paper immediately.
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Data were analyzed using a thematic analysis approach. The coding process followed a four-step sequence to eliminate theoretical bias and focus on raw data:

- Open Coding: Raw transcriptions and written tests were broken into meaningful units (e.g., "S1 pausing to look back at Task 1").
- Axial Coding: Codes were clustered around specific behavioral properties (e.g., *Self-Correction*, *Formula Hunting*, *Contextual Block*).
- Selective Coding: Data were integrated into the primary themes: Planning (P), Monitoring (M), and Evaluation (E).
- Triangulation: Test scripts, transcript data, and observations were cross-validated before drawing final conclusions.

3. FINDINGS AND DISCUSSION

3.1 Findings

The qualitative analysis revealed distinct variances in how the three subjects (S1, S2, and S3) engaged their metacognitive capacities. Crucially, across all levels, the component of Evaluation was found to be the most unevenly developed and fragile.

3.1.1 Comparative Analysis across Student Capabilities

To provide a consolidated view of the empirical data, Table 3 contrasts the explicit behaviors exhibited by the high, medium, and low-ability students during the problem-solving tasks.

Table 3. Cross-Case Comparison of Subject Metacognitive Behaviors

Metacognitive Process	High Subject (S1)	Medium Subject (S2)	Low Subject (S3)
Planning (P)	Re-reads task twice; extracts explicit variables; maps the proportional model instantly.	Extracts main figures immediately; writes down a basic Cross-Multiplication formula.	Jumps straight to multiplication/division without establishing ratios.
Monitoring (M)	Mentally compares units; checks if basket count correlates logically with time.	Follows algorithmic steps sequentially; does not check if the answer direction is logical.	Gets stuck when numbers do not divide evenly; expresses confusion without changing path.
Evaluation (E)	Recalculates via an alternative method (unitary rate) to confirm the final figure.	Assumes correctness because "the formula matches the classroom notes."	Does not look back; fails to realize that workers should increase, not decrease.

3.1.2 Subject Empirical Profiles: Evidences from Scripts and Transcripts

a. High Metacognitive Ability (S1)

Subject S1 showed an integrated, self-regulating mechanism. On Task 1, the student's written response clearly established the ratio:

$$\frac{15 \text{ basket}}{5 \text{ hari}} = \frac{40 \text{ basket}}{x \text{ hari}}$$

S1 executed the cross-multiplication flawlessly

$$15x = 240$$

$$x = 16$$

During the semi-structured interview, S1 revealed active self-monitoring:

R : "What did you think when you first read the basket problem?"

S1 : "I first checked if it was direct or inverse. More baskets mean more days needed, so it must be direct. I

wrote down  to visualize the balance."

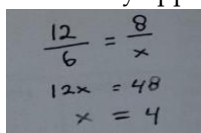
R : "How did you make sure 16 days was correct?"

S1 : "I divided 15 by 3 to see what 2 days gets us. 5 baskets in 2 days. Then I scaled it up to 40 baskets. It matched exactly."

This data confirms that S1 does not rely solely on memorized procedures. Instead, S1 planning serves as a mental platform that makes subsequent monitoring smooth. This finding supports the structural model of Aditomo and Klieme (2020), which states that active cognitive engagement yields more adaptive strategy execution.

b. Medium Metacognitive Ability (S2)

Subject S2 demonstrated adequate procedural execution but partial monitoring. On Task 2 (Inverse Proportion), S2's written solution mistakenly applied a direct proportion format:

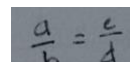


S2 concluded that 4 workers were needed, failing to notice that fewer days should require *more* workers.

The interview clip illustrates this metacognitive gap:

R : "Look at your answer for the road project. It says 4 workers. Does that make sense?"

S2 : "Yes, because I followed the cross-multiplication steps we learned



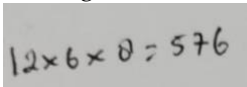
R : "Did you check your work after finishing?"

S2 : "No, I checked the multiplication on the side of the paper, and the math was correct, so I thought the whole answer was right."

This excerpt highlights a critical qualitative insight: procedural confidence often masks conceptual errors. S2 monitored the calculation arithmetic but failed to monitor the conceptual logic. This extends the work of Wijaya et al. (2019) by demonstrating that middle-tier students treat contextual word problems as mere number-crunching exercises, ignoring the real-world boundaries of the problem.

c. Low Metacognitive Ability (S3)

Subject S3 exhibited fragmented and erratic behaviors. On Task 2, S3 simply multiplied the numbers

given in the text: . There was no visible planning framework on the script.

The interview transcription emphasizes the complete absence of reflective thinking:

R : "Can you explain why you multiplied all these numbers together?"

S3 : "I didn't know what to do. I just saw three numbers and multiplied them hoping to get the total."

R : "Did you re-read the problem when you got stuck?"

S3 : "No, I just wanted to fill the empty space. I don't know how to check if it's right."

For S3, problem-solving is an act of guesswork. This confirms the warnings of Sumaryanta et al. (2023) regarding the severe lack of higher-order strategic foundations in lower-performing cohorts.

3.1.3 Why Evaluation is the Weakest Link

A major finding of this study is that Evaluation is consistently the weakest metacognitive component across all three levels. Even when a student achieves procedural correctness (as S1 did or as S2 did in basic tasks), reflective evaluation remains underdeveloped.

Our qualitative data suggests two primary causes for this phenomenon:

- The "Product-Over-Process" Classroom Culture: Students are conditioned to treat the final numerical box as the termination point of learning. Once an answer is produced, cognitive effort drops to zero.
- Affective Avoidance: As seen in S2' s interview, re-checking is avoided because it introduces cognitive dissonance (the fear of finding a mistake and having to redo the work).

This finding challenges the traditional assumption that a correct answer implies reflective mastery. Evaluation requires high-level self-assessment (Magiera & Zawojewski, 2018). Without explicit external prompts, students rarely initiate this process on their own.

3.1.4 Concrete Pedagogical Implications for Proportional Reasoning

Given that proportional reasoning acts as a gateway to advanced algebraic thinking, mathematics instruction must shift from teaching formulas to teaching cognitive management. Instead of giving students direct answers, teachers should use metacognitive scaffolding prompts during lessons.

Table 4. Concrete Classroom Scaffolding Interventions

Problem Stage	Traditional Prompt (Procedural)	Metacognitive Prompt (Reflective)	Expected Student Outcome
Initial Phase	"What formula should you use here?"	"Before calculating, if the number of baskets goes up, should your answer be larger or smaller than 6 days? Why?"	Prevents mechanical formula matching; forces qualitative planning.
Mid-Process	"Keep doing the cross-multiplication."	"Look at the unit rate you just found. What does that number actually mean in terms of a single worker's daily output?"	Breaks algorithmic blindness; promotes active monitoring.
Final Phase	"Write down the final answer unit."	"Imagine you are the contractor. Does it make logical sense that fewer workers can finish the road faster? How can you prove your answer is reasonable?"	Overcomes the weakness in evaluation; bridges math with real-world contexts.

By embedding these specific reflective prompts into problem-based learning designs, teachers can help students build a more connected metacognitive system. This approach directly addresses the uneven cognitive development identified in this study.

3.2 Discussion

This study provides a nuanced understanding of how students' metacognitive processes differ when solving direct and inverse proportion problems. The findings demonstrate that metacognitive regulation is not a uniform construct but consists of interconnected processes of planning, monitoring, and evaluation that develop at different rates across learners. Students with high metacognitive ability exhibited coherent regulation throughout problem solving by carefully identifying relevant information, selecting appropriate strategies, monitoring the consistency of their reasoning, and verifying the plausibility of their final answers. In contrast, students with medium metacognitive ability demonstrated adequate planning and procedural execution but insufficient monitoring of conceptual relationships and limited evaluation of solution validity. Students with low metacognitive ability relied predominantly on trial-and-error strategies, showed minimal evidence of planning or self-monitoring,

and rarely reflected on the correctness of their responses. These findings reinforce Flavell's (1979) proposition that successful problem solving depends not only on cognitive knowledge but also on the regulation of one's own thinking processes. They also support Efklides' (2020) argument that metacognitive regulation enables learners to adapt their reasoning in response to task demands rather than merely applying memorized procedures.

A notable contribution of this study is the identification of **evaluation** as the weakest metacognitive component across all ability levels. Even students who produced correct solutions frequently failed to verify their reasoning using alternative strategies or to evaluate whether their answers were logically consistent with the problem context. This finding suggests that procedural success should not automatically be interpreted as evidence of reflective understanding. Instead, students often terminate their cognitive effort immediately after obtaining a numerical answer, indicating that evaluation remains an underdeveloped aspect of mathematical problem solving. This observation is consistent with Magiera and Zawojewski (2018), who argued that reflective verification represents one of the highest levels of metacognitive functioning because it requires learners to critically examine both the process and outcome of their reasoning. Likewise, Nelson and Narens (1990) emphasized that effective self-regulation depends on continuous monitoring and feedback between cognitive performance and metacognitive control, a process that was largely absent among the lower-ability participants in this study.

The differences observed among the three participants also demonstrate that proportional reasoning demands considerably more than procedural competence. Students with high metacognitive ability consistently interpreted the relationships between quantities before selecting a mathematical strategy, whereas medium- and low-ability students tended to associate proportional problems with familiar computational procedures without first considering whether the relationship was direct or inverse. Such findings indicate that conceptual interpretation precedes successful procedural execution. This extends previous work by Wijaya et al. (2019), who reported that many students experience difficulties in contextual mathematical problems because they focus primarily on numerical manipulation while overlooking the meaning embedded within the problem situation. The present study further suggests that these conceptual difficulties are closely intertwined with deficiencies in metacognitive regulation, particularly the ability to monitor whether an emerging solution remains consistent with contextual information.

Another important finding concerns the interaction among planning, monitoring, and evaluation. Rather than functioning as independent skills, these three components operated as an integrated regulatory system in which weaknesses in one component affected the effectiveness of subsequent processes. Students who failed to establish an appropriate plan frequently encountered difficulties during monitoring and consequently lacked sufficient information to conduct meaningful evaluation. Conversely, participants who carefully analyzed the problem before initiating calculations demonstrated greater flexibility in identifying errors and validating their solutions. This interconnected pattern aligns with contemporary perspectives on metacognition, which conceptualize self-regulation as a dynamic interaction among planning, monitoring, and evaluation rather than as isolated cognitive activities (Callan & Cleary, 2018; Efklides, 2020). Accordingly, mathematics instruction should encourage students to regulate their thinking continuously throughout problem solving instead of treating reflective evaluation as an optional activity performed only after completing calculations.

The pedagogical implications of these findings are particularly relevant for mathematics classrooms. Traditional instruction often emphasizes procedural fluency and obtaining correct answers, inadvertently reinforcing a product-oriented learning culture. The present findings suggest that such instructional practices may contribute to students' limited engagement in reflective evaluation. Teachers should therefore integrate explicit metacognitive scaffolding into classroom activities by encouraging students to predict the direction of proportional relationships before calculating, justify the strategies they select, monitor whether intermediate results remain logically consistent, and verify final answers using alternative approaches or contextual reasoning. These practices are consistent with research demonstrating that explicit metacognitive instruction enhances

mathematical reasoning and promotes self-regulated learning (Aditomo & Klieme, 2020; Susanti et al., 2023). Embedding reflective questioning within problem-based learning or inquiry-oriented mathematics instruction may therefore help students develop stronger conceptual understanding while simultaneously strengthening their metacognitive regulation.

Finally, this study contributes to the growing literature on metacognition in mathematics education by providing an in-depth qualitative examination of metacognitive processes within the specific context of direct and inverse proportion problems. Whereas previous studies have predominantly measured metacognitive awareness quantitatively, this research illustrates how planning, monitoring, and evaluation manifest during authentic problem-solving activities and how these processes vary across students with different metacognitive profiles. Although the findings are based on a limited number of participants and therefore are not intended for statistical generalization, they provide rich evidence supporting the view that metacognitive regulation develops unevenly and substantially influences the quality of mathematical reasoning. Future studies employing larger samples or mixed-methods designs could further investigate instructional interventions that specifically strengthen evaluation, thereby promoting more reflective and conceptually grounded mathematical problem solving.

4. CONCLUSION

This study reveals that junior high school students' metacognitive processes in solving direct and inverse proportion problems differ across ability levels, particularly in how they plan, monitor, and evaluate their problem-solving strategies. A key finding is that evaluation emerges as the weakest metacognitive component across all proficiency levels, even when students are able to arrive at correct answers. This indicates that successful problem solving is not solely determined by procedural accuracy, but also by students' ability to reflect on and verify their reasoning. From a theoretical perspective, this study contributes to metacognitive theory by demonstrating that planning, monitoring, and evaluation do not develop uniformly. Instead, they function as an interdependent system that varies according to students' overall cognitive profiles. Weaknesses in one component particularly evaluation can limit the overall effectiveness of problem solving. However, given the qualitative nature of this study and the limited sample size of three subjects, these conclusions are context-specific and should not be over-generalized to broader student populations. In terms of practical implications, mathematics instruction should explicitly integrate metacognitive strategies into classroom practice. Teachers are encouraged to support students in developing structured planning, consistent monitoring, and especially reflective evaluation through guided questioning and step-by-step scaffolding. Additionally, the use of problem-based learning can provide meaningful and authentic contexts for students to develop deeper conceptual understanding in solving proportion problems. Future research should involve larger and more diverse samples using mixed-methods approaches to explore the generalizability of these patterns. Further studies could also investigate the effectiveness of specific instructional interventions designed to strengthen evaluation as the most underdeveloped component across different mathematical topics and educational level

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