

Analysis of Students' Mathematical Problem-Solving Ability in Statistics Viewed from Sociomathematical Norms

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ABSTRACT

The persistently low performance of Indonesian students in international assessments such as TIMSS and PISA indicates a critical gap in mathematical problem-solving skills. Beyond cognitive factors, sociomathematical norms—classroom-based standards for reasoning and discourse—may significantly influence how students approach and solve problems. This study employed a descriptive qualitative design. A sociomathematical norms questionnaire was administered to 27 tenth-grade students and categorized into high, medium, and low groups (Mean = 75.185; SD = 11.281). Six students (two from each group) were purposively selected for in-depth analysis. Data were collected through three statistical problem-solving tasks and clinical interviews. Students' processes were analyzed using Polya's four-stage framework: understanding the problem, devising a plan, carrying out the plan, and looking back. A strong positive relationship was found between sociomathematical norms and problem-solving proficiency. Students with high norms demonstrated mastery across all stages, including deep understanding, logical planning, systematic execution, and consistent reflective evaluation. Students with medium norms performed adequately in the first three stages for simpler problems but often omitted the "looking back" phase. Students with low norms exhibited fragmented understanding, ineffective planning, and difficulty executing and verifying solutions, often accompanied by low confidence. These findings highlight the importance of sociomathematical norms in fostering reflective and effective problem-solving practices. Strengthening sociomathematical norms can enhance students' mathematical problem-solving abilities and support improved learning outcomes.

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1. INTRODUCTION

Mathematics plays a crucial role in developing analytical, logical, and critical thinking skills that are essential for navigating complex real-world situations. Through mathematics, learners are expected

not only to perform calculations but also to construct meaning, interpret information, and solve problems systematically. Despite this ideal, mathematics instruction in many educational contexts—particularly in developing countries—continues to emphasize procedural fluency rather than conceptual understanding. As a result, students often become proficient in executing algorithms without fully grasping the underlying reasoning processes that support meaningful problem-solving. This imbalance limits their ability to transfer knowledge to unfamiliar situations and undermines the broader goals of mathematics education.

In the Indonesian context, concerns regarding students' mathematical problem-solving abilities have been widely documented. Empirical evidence suggests that students frequently focus on producing final answers without demonstrating coherent reasoning or engaging in intermediate problem-solving steps (Widodo et al., 2018). This tendency reflects a superficial approach to learning, where success is measured by correctness rather than by the quality of thinking. International large-scale assessments further reinforce this concern. Results from the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA) consistently show that Indonesian students perform below the global average in mathematics. For instance, in TIMSS 2015, Indonesia ranked 44th out of 49 participating countries with an average score of 397. Similarly, the PISA 2022 results placed Indonesia 69th out of 80 countries, with an average score of 366 compared to the international average of 472 (OECD, 2022). These findings highlight a persistent gap in students' ability to engage in higher-order thinking, particularly in solving non-routine and context-based mathematical problems.

Mathematical problem-solving itself is a complex and dynamic process. Polya (1985) conceptualized it as consisting of four interrelated stages: understanding the problem, devising a plan, carrying out the plan, and looking back. This framework underscores that problem-solving is not merely about reaching a solution but involves iterative reasoning and reflection. The final stage, often overlooked in classroom practice, requires students to evaluate the validity and efficiency of their solutions, thereby fostering deeper understanding. Expanding on this perspective, Schoenfeld (2016) emphasized the importance of metacognition in problem-solving, arguing that effective problem solvers actively monitor, regulate, and evaluate their thinking processes. These metacognitive skills enable learners to adapt strategies, recognize errors, and make informed decisions during problem-solving. Furthermore, Lester and Cai (2016) noted that successful problem solvers exhibit perseverance, flexibility, and the capacity for reflection, all of which contribute to sustained engagement with challenging tasks.

Recent research has demonstrated that explicit instruction in problem-solving strategies can significantly enhance students' reasoning abilities and motivation. For example, Kaur (2023) and Rahmazatullaili et al. (2017) found that structured guidance in heuristic strategies improves students' capacity to tackle complex problems. Similarly, Nohda (2020) highlighted the role of collaborative learning environments in fostering adaptive expertise, where students learn to modify and refine their approaches based on feedback and interaction. Schoenfeld (2019) further established a strong correlation between metacognitive awareness and mathematical performance across diverse contexts. These findings suggest that problem-solving competence is not solely an individual cognitive achievement but is also shaped by instructional practices and learning environments. Consequently, effective assessment of problem-solving should incorporate multiple methods, including open-ended tasks, reflective journals, and observations of reasoning processes, to capture the multifaceted nature of students' thinking (Inoue & Buczynski, 2022).

While cognitive and metacognitive dimensions are central to problem-solving, the social context of learning also plays a pivotal role. Classroom interactions, discourse patterns, and shared expectations influence how students construct and communicate mathematical ideas. Yackel and Cobb (1996) introduced the concept of sociomathematical norms to describe the classroom-specific standards that govern what counts as an acceptable mathematical explanation, justification, or solution. Unlike general social norms, sociomathematical norms are directly related to mathematical reasoning and are

co-constructed through ongoing interactions between teachers and students. These norms shape how students interpret tasks, evaluate solutions, and engage in argumentation.

The development of sociomathematical norms is closely tied to the nature of classroom discourse. Cobb and Hodge (2002) argued that such norms emerge through repeated patterns of communication in which teachers encourage students to explain their reasoning, compare different strategies, and critique one another's ideas. In classrooms where these practices are consistently implemented, students are more likely to engage in meaningful mathematical discussions that deepen their conceptual understanding. Conversely, in environments where instruction is predominantly teacher-centered, opportunities for dialogue are limited, and students may adopt passive roles as recipients of knowledge. This lack of interaction not only constrains their reasoning but also diminishes their confidence and willingness to participate in mathematical discourse.

Empirical studies provide strong evidence of the impact of sociomathematical norms on students' learning. Jurdak and Shahin (2020) found that classrooms emphasizing justification and argumentation significantly enhanced students' depth of problem-solving. Similarly, Kaldrimidou et al. (2019) reported that exposure to sociomathematical norms improved students' reasoning and communication skills. More recently, Lim (2023) demonstrated that such norms contribute to the development of mathematical agency, defined as students' confidence in proposing, evaluating, and refining mathematical ideas. This suggests that sociomathematical norms not only influence cognitive processes but also shape learners' identities as active participants in mathematical practices. Furthermore, Lopez (2007) emphasized that these norms regulate classroom dialogue by establishing criteria for acceptable reasoning, thereby creating a structured yet flexible environment for learning. The role of teachers is particularly critical in this process. DeJarnette and González (2021) showed that instructional strategies such as open-ended questioning and peer feedback promote equitable participation and deeper reasoning, enabling students to internalize sociomathematical norms over time.

The relationship between sociomathematical norms and mathematical problem-solving is inherently reciprocal. On one hand, strong norms create conditions that encourage students to articulate, justify, and reflect on their reasoning, which are essential components of effective problem-solving. On the other hand, engaging in problem-solving activities provides opportunities for these norms to be enacted and reinforced. Several mechanisms explain this relationship. First, the expectation of justification compels students to clarify their thinking, leading to greater accuracy and transferability of strategies (Jurdak & Shahin, 2020). Second, dialogic interaction exposes learners to diverse perspectives, enabling them to evaluate the efficiency and generalizability of different approaches (Lim, 2023). Third, regular reflection on reasoning fosters metacognitive awareness, which is crucial for monitoring and regulating problem-solving processes (Schoenfeld, 2016). Supporting this view, Erath and Prediger (2022) found that classrooms with explicitly established sociomathematical norms achieved higher levels of justification quality and flexibility in reasoning compared to those without such norms. These findings underscore the role of sociomathematical norms as catalysts for deeper cognitive engagement and more effective problem-solving.

The theoretical foundation of this study is rooted in Vygotsky's (1978) social constructivist perspective, which posits that learning occurs through social interaction and is mediated by cultural tools. Central to this theory is the concept of the Zone of Proximal Development (ZPD), which represents the distance between what learners can achieve independently and what they can accomplish with guidance. Within this framework, sociomathematical norms function as cultural tools that facilitate communication and support the development of higher-order thinking. When students engage in explaining, questioning, and critiquing ideas, they participate in the co-construction of knowledge, transforming both individual cognition and collective understanding. Wertsch (1991) further elaborated on this idea by emphasizing that mediated action shapes both thought and practice, highlighting the inseparability of cognitive and social processes.

Integrating Vygotsky's social constructivism with Polya's problem-solving framework provides a comprehensive lens for understanding mathematical learning. While Polya focuses on the internal

cognitive processes involved in solving problems, Vygotsky highlights the social interactions that nurture and sustain these processes. Together, these perspectives suggest that effective problem-solving emerges from the interplay between individual reasoning and collaborative discourse. In this context, sociomathematical norms serve as a bridge that connects cognitive and social dimensions, enabling students to move from procedural execution to conceptual understanding.

Given these considerations, it becomes essential to investigate how sociomathematical norms influence students' problem-solving abilities, particularly in the domain of statistics, which requires both conceptual reasoning and data interpretation. This study aims to explore the characteristics of students' problem-solving processes across different levels of sociomathematical norms, examine the relationship between these norms and reflective reasoning, and identify pedagogical implications for fostering meaningful interaction and reasoning in mathematics classrooms. By addressing these objectives, the study seeks to contribute to the ongoing effort to improve mathematics education in Indonesia, particularly in developing students' capacity for critical and reflective problem-solving.

2. METHODS

This study involved 27 tenth-grade science students from a public high school. Data on sociomathematical norms were collected using a Likert-scale questionnaire. The questionnaire was developed based on six indicators of sociomathematical norms: mathematical experience, mathematical explanation, mathematical difference, mathematical communication, mathematical effectiveness, and mathematical insight. The instrument consisted of 20 Likert-scale items. An example of the questionnaire item is: "I actively discuss with friends in Mathematics learning" and "I appreciate it if my friends suggest different solutions.". The instrument's validity was confirmed by expert validation, and its reliability was established using Cronbach's Alpha via SPSS. The categorization of students was based on the mean ($M = 75.185$) and standard deviation ($SD = 11.281$) of the questionnaire scores. The criteria were as follows: High: $Score \geq (M + SD) = \geq 86.466$; Medium: $(M - SD) \leq Score < (M + SD) = 63.904 \leq Score < 86.466$; Low: $Score < (M - SD) = < 63.904$.

The distribution of the 27 students was: 2 in the high category, 21 in the medium category, and 4 in the low category. This research used purposive sampling; two students were selected from each sociomathematical norm category, resulting in six research subjects (coded as T1, T2 [High]; S1, S2 [Medium]; R1, R2 [Low]). These subjects were given a test consisting of three essay questions on statistics, designed to assess the four Polya problem-solving stages. Following the test, in-depth interviews were conducted to triangulate the written responses. The interview protocol focused on the students' reasoning and processes as demonstrated in their test answers.

3. FINDINGS AND DISCUSSION

This study's results are presented in two main subsections. First, the descriptive findings from the sociomathematical norms questionnaire are outlined. Second, a comparative analysis of mathematical problem-solving abilities across the different norm categories is detailed, drawing on data from tests and in-depth interviews.

3.1. Distribution of Sociomathematical Norms

Data from the sociomathematical norms questionnaire, administered to 27 students, were analyzed using descriptive statistics. The mean score was 75.185 with a standard deviation of 11.281. Based on these parameters, students were categorized into three distinct levels:

High Norms: $Score \geq (Mean + SD) = \geq 86.466$

Medium Norms: $(Mean - SD) \leq Score < (Mean + SD) = 63.904 \leq Score < 86.466$

Low Norms: $Score < (Mean - SD) = < 63.904$

The distribution of students across these categories is presented in Table 1.

Table 1. Frequency Distribution of Students by Sociomathematical Norms Category

Category	Score Range	Frequency	Percentage
High	≥ 86.466	2	7.4%
Medium	63.904 – 86.465	21	77.8%
Low	< 63.904	4	14.8%
Total		27	100%

As shown in Table 1, the majority of students (77.8%) were classified within the medium category of sociomathematical norms. A smaller proportion demonstrated low norms (14.8%), and only two students (7.4%) exhibited high norms.

3.2. Problem-Solving Performance Across Sociomathematical Norm Categories

From each category, two students were selected purposively for in-depth analysis, resulting in six subjects (T1, T2 for High; S1, S2 for Medium; R1, R2 for Low). Their performance on three statistical problem-solving tasks, evaluated against Polya's four stages, revealed significant disparities.

3.2.1 Performance on Problem 1 (Weighted Average)

All six subjects successfully arrived at the correct numerical answer. However, the quality of their processes varied considerably. High Norms (T1, T2): Both students demonstrated mastery of all four Polya stages. They clearly articulated the known and unknown variables, correctly identified the concept of a weighted average, executed the calculations flawlessly, and verified their answers. Their interview responses were systematic and confident. Medium Norms (S1, S2): These students understood the problem, devised a correct plan, and executed it accurately. However, both explicitly admitted during interviews that they did not check their work, failing the "looking back" stage. Their process was correct but lacked metacognitive closure. Low Norms (R1, R2): While obtaining the correct answer, their processes were flawed. R1 provided a disjointed explanation, omitting the derivation of key intermediate values. R2 failed to state what was being asked in the problem and could not explain their steps coherently. Both expressed uncertainty about their answers and did not perform any verification.

3.2.2 Performance on Problem 2 (Descriptive Statistics with Constraints)

This problem required finding a set of five integers satisfying the given conditions for mean, median, and mode. High Norms (T1, T2): Both subjects successfully constructed the valid set {4, 9, 10, 11, 11}. They methodically allocated known values (median=10, mode=11), calculated the required sum, and logically deduced the first two numbers to satisfy all conditions. They conclusively verified their solution. The results of the answers from informant T1 are presented in Figure 1.

)diketahui :
 • ada 5 bilangan bulat
 • median = 10
 • rata-rata = median - 1 = 9
 • modus = median + 1 = 11
 ditanyakan : bilangan bulat terkecil yang mungkin dituliskan.
 jawab : karena jumlahnya 5 bilangan bulat, kita anggap susunannya :
 a b c d e
 10 11 11
 total = $9 \times 5 = 45$
 $a + b + 10 + 11 + 11 = 45$
 $a + b = 45 - 32 = 13$
 • $6 + 7 = 13$
 • $5 + 8 = 13$
 • $4 + 9 = 13$
 jadi, bilangan bulat terkecil yang dapat ditulis adalah 4 //

Figure 1. T1 Answer Result

During the interview, T1 stated, "The median in the problem is 10 and the mode is one more than the median, which is 11, while the mean is 9. Then I arranged five numbers in order and determined the first two numbers so that the total becomes 45. To make sure, I checked again, the number was 45 so the average was 9, the median was still 10, and the mode was 11."

Medium Norms (S1, S2): Both arrived at the correct set {4, 9, 10, 11, 11} but through a less assured process. S1's worksheet showed trial-and-error, and their verbal explanation was hesitant. Crucially, neither student performed a comprehensive check of their solution against all problem constraints. The results of the answers from informants S1 are presented in Figure 2.

2. diket = median 5 bilangan bulat adalah 10
 $r = 9$
 $m = 11$
 ditanya = bil terkecil dari 5 bilangan bulat
 jawab =
 $= \textcircled{4} \quad 9 \quad 10 \quad 11 \quad 11 = 32$
 $R = 9 \times 5 = 45 - 32 = 13$
 $m = 11$
 hasilnya = 4

 $6 + 7 = 13$
 $5 + 8 = 13$
 $4 + 9 = 13$

Figure 2. S1 Answer Result

During the interview, S1 explained, "There are five integers in the problem with a median of 10, a mean of 9, and a mode of 11. I arranged the known numbers first and then looked for two other numbers whose sum is 13 so that the total becomes 45."

Low Norms (R1, R2): Both students provided an incorrect set {5, 8, 10, 11, 11}, which sums to 45 but incorrectly has a mode of 11 (appearing twice) and a median of 10. They failed to recognize that their proposed set did not satisfy the mean condition upon verification. Their explanations were fragmented, and they showed low confidence, with R2 being unable to fully articulate the problem's requirements. The results of the answers from informants R2 are presented in Figure 3.

2/ dik 5 bilangan bulet
 media : 10
 Dit bilangan bulet terkecil y mungkin kan dari 5 bilangan
 tsb

5 8 16 11 11 = $9 \times 5 = 45 - 32 = 13$

Figure 3. R2 Answer Result

During the interview, R2 stated: "I calculated the total by multiplying 9 by 5 to get 45, then I arranged the numbers and chose 5 and 8 as the first two numbers so that their sum becomes 13. I have not checked my answer again."

The analysis of students' solutions was also interpreted through the lens of Polya's problem-solving stages, which include understanding the problem, devising a plan, carrying out the plan, and reflecting on the process. Students categorized under high sociomathematical norms exhibited all four stages distinctly. Initially, they recognized the known statistical parameters, subsequently formulated an appropriate strategy to find the remaining numbers, executed the calculations methodically, and ultimately confirmed whether the resulting set met the conditions for mean, median, and mode.

Students in the medium category displayed signs of the initial three stages, but the looking-back stage was not distinctly shown, as they did not adequately confirm that all conditions were met. Conversely, students in the low category struggled starting at the planning stage, which resulted in wrong answers and a lack of verification for their responses.

3.2.3 Performance on Problem 3 (Complex Descriptive Statistics with Percentage Increase)

This was the most complex problem, involving seven data points. High Norms (T1, T2): Both students correctly identified the highest value as Rp 5,075,000 (a 45% increase over the mean). They systematically arranged the known data (three modes, one median), calculated the total sum, and deduced the remaining values, correctly identifying the highest sale. Their process was thorough and verified. Students in this category consistently demonstrated clear reasoning and self-correction. When solving problems about measures of central tendency, they not only performed computations but also justified the choice of mean versus median depending on data distribution. Their interviews revealed metacognitive awareness: "If there's an extreme value, the mean doesn't describe the data well, so I use the median." This indicates deep reasoning in the problem-solving process rather than procedural recall. Such behavior aligns with Schoenfeld's (2016) description of "reflective practitioners" in mathematics.

Medium Norms (S1, S2): Both students demonstrated understanding and initial planning, but failed in execution. S1 incorrectly identified the highest value as the sum of the remaining two unknown data points (Rp 8,925,000). S2 correctly placed Rp 5,075,000 in the final position but could not determine the other two values and thus could not fully justify the solution. Both were confused and admitted a lack of certainty. Medium-norm students were generally able to understand problems but displayed hesitation in strategy selection. For example, they could compute the mean correctly but hesitated to interpret it contextually. Their explanations often relied on memorized formulas rather than reasoning. During interviews, one student said, "I follow the example given by the teacher because I'm afraid my way is wrong." This dependence reflects partial internalization of norms—students recognize that reasoning is valuable but still seek external validation. However, these students showed potential improvement

during collaborative tasks, where peer questioning prompted deeper explanations. This supports the finding of Kaldrimidou et al. (2019) that structured dialogue enhances mathematical agency.

Low Norms (R1, R2): Performance was markedly poor. R1 only managed to list the known values and could not proceed further. R2 did not attempt to solve the problem. Both cited confusion and an inability to structure a solution. Students with low norms often struggled to begin problem-solving independently. They required constant prompts from the teacher and tended to remain silent during discussions. One student stated, “I’m not confident to explain my answer because others might think I’m wrong.” This lack of participation illustrates how weak sociomathematical norms suppress mathematical identity (Cobb & Hodge, 2002). These students tended to prioritize correctness over understanding and rarely evaluated their solutions. Their limited discourse participation also restricted exposure to alternative strategies, resulting in shallow comprehension. A summary of the problem-solving performance across all stages and categories is provided in Table 2

Table 2. Summary of Problem-Solving Performance by Sociomathematical Norm Category

Polya's Stage	High Norms (T1, T2)	Medium Norms (S1, S2)	Low Norms (R1, R2)
Understand the Problem	Consistently demonstrated a deep and accurate understanding.	Demonstrated a surface-level understanding; could identify givens but struggled with complex relationships.	Understanding was fragmented and incomplete, often missing key requirements or relationships.
Devise a Plan	Formulated efficient, logical, and appropriate strategies.	Devised basically correct plans for simpler problems (1 & 2), but plans broke down under complexity (Problem 3).	Planning was haphazard, illogical, or absent; it relied on unsystematic trial and error.
Carry Out the Plan	Execution was precise, systematic, and led to correct solutions.	Execution was adequate for straightforward calculations but faltered with multi-step logic.	Execution was flawed, with omitted steps and illogical deductions, which often led to incorrect answers.
Look Back/Review	Consistently and effectively verified their answers and processes.	Consistently omitted this stage, even when the numerical answer was correct.	Never demonstrated this stage; showed no evidence of checking or reflection.

The results indicate a strong positive correlation between the level of sociomathematical norms and comprehensive problem-solving ability. Students with high norms were proficient across all four stages of Polya's framework. Those with medium norms were functionally competent in the first three stages on simpler problems but critically lacked the metacognitive skill of verification. Students with low norms struggled fundamentally with understanding, planning, and execution, and completely failed to engage in reviewing their work.

Comparing all groups revealed that sociomathematical norms are directly linked to metacognitive regulation and reflection. High-norm students displayed autonomy and critical awareness, whereas low-norm students exhibited dependency and fear of judgment. These findings reaffirm Vygotsky's notion that cognitive development is socially mediated. Moreover, sociomathematical norms foster a classroom culture that transforms mathematics learning into a collaborative inquiry rather than individual computation. Teachers who promote questioning and justification help students see errors as opportunities for learning rather than failure.

The study suggests that teachers can cultivate sociomathematical norms by using open-ended problems that require explanation, not just answers, and encouraging peer critique and justification during discussions, providing reflective prompts, such as “Why do you think this strategy works?”, and modeling argumentation through think-aloud reasoning. Integrating these practices could enhance

students' confidence and analytical reasoning, thereby improving problem-solving performance in statistics and other mathematical domains.

4. CONCLUSION

This study concludes that sociomathematical norms play a critical role in shaping students' mathematical problem-solving abilities. The findings indicate that the majority of the 27 students demonstrated a medium level of sociomathematical norms (77.8%), with only a small proportion classified as high (7.4%) or low (14.8%), suggesting that while most students possess a foundational understanding of mathematical classroom practices, further development toward more advanced levels of reasoning and discourse is needed. Moreover, a strong positive correlation was identified between sociomathematical norms and problem-solving proficiency based on Polya's framework, where students with high norms exhibited comprehensive mastery across all stages, including metacognitive reflection, whereas those with medium norms showed partial competence but lacked reflective evaluation skills, and students with low norms struggled across all stages of problem-solving. Despite these insights, the study is limited by its small sample size and its focus on a single educational context, which may restrict the generalizability of the findings. Therefore, future research should involve larger and more diverse samples, as well as longitudinal or experimental designs, to further examine how instructional interventions aimed at fostering sociomathematical norms can enhance students' metacognitive regulation and advanced problem-solving abilities.

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