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Teachers' Multiple Representations on Teaching Fractions at Elementary School: A Commognitive Framework

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ABSTRACT

This research explores how teachers utilize verbal, symbolic, pictorial, and real-life representations using a commognitive framework to support student learning. This research uses a qualitative approach with an exploratory descriptive design. The selection of research subjects was carried out through purposive sampling which was modified to achieve the research objectives. The research focuses on elementary school teachers who have education diplomas and more than 15 years of teaching experience. In this research, our data is in the form of learning video recording data. The collected data was analyzed using the cognition framework and the Lesh Model of multiple representations. The results of this research show two representations that teachers always apply to each commognitive component, namely symbolic representation and real-life representation. Symbolic representations are often used by teachers to solve mathematical problems, especially word problems. Likewise, real life representations are often used because they apply events or occurrences in everyday life. The use of diverse representations has the potential to increase students' understanding and ability to apply fraction concepts in real life.

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1. INTRODUCTION

Globalization and technological progression in the 21st century provide particular challenges for students (Nithyanantham et al., 2019; Scott, 2000). Students are expected to understand and overcome everyday life problems with strong evaluation capacities, such as problem-solving (Hidayah et al., 2023). Education plays a vital role in preparing students to face those phenomena (Inayati, 2022). The need to prepare a better generation is reflected in the several curriculum changes in the world of education. Currently, in Indonesia, we implement a Kurikulum Merdeka (independent curriculum) as a guide in the learning process, following the needs of students. The Kurikulum Merdeka provides a more diverse intracellular, so students are expected to have more time to understand the concepts and enhance their competencies (Ramadani & Desyandri, 2022; Sunarni & Karyono, 2023)

Students can properly learn mathematics after they construct their own understanding. Meanwhile, this understanding of mathematical concepts requires them to examine, represent, transform, solve, apply, prove, and communicate (NCTM, 2000; Raj Acharya, 2017). Likewise, teachers' excellent communication will support the success of students' learning. In addition to teaching primary mathematic material, teachers should also train students' skills in applying mathematical concepts in solving complex problems, requiring other branches of knowledge (Rahayuningsih et al., 2021; Sa'dijah et al., 2023). The available alternative for teachers to communicate theirearning is commognitive (Sfard, 2008). For teachers, commognitive can be used as a means for developing their competencies during learning (Zayyadi et al., 2020). As an approach that emphasizes the importance of language and communication in learning, commognitive focuses on commognitive processes and the use of communication in learning processes. The commognitive component is determined based on word use, visual mediator, routine, and narrative (Zayyadi et al., 2020). A description of the commognitive framework is presented in Table 1.

Table 1. Description of Commognitive Framework

Components	Descriptions
Word Use	The use of words or sentences in learning mathematics
Visual Mediators	Objects used in learning mathematics
Narrative	The mathematical language in mathematics learning including the definitions, theorems, and facts.
Routines	The process of rules that describe patterns in learning mathematics

Previous research on commognitive was carried out by (Sfard, 2008), reporting the potential of commognitive as an alternative in learning communication. In addition, that study also serves as an empirical study describing theory as a method (Kim et al., 2017) used in solving mathematical problems (Zayyadi et al., 2020). A previous study described the teacher's ability to learn mathematics based on commognitive components (Zayyadi et al., 2022). This research specifically examines teachers' efforts to explain mathematical concepts through four commognitive communications, namely the use of words, visual mediators, routines and narratives. Commognitive provides an overview of mathematical cognitive-communication and content in the learning carried out. Therefore, the use of a commognitive is very important to apply in mathematics learning.

In addition, by associating previous mathematical knowledge and new knowledge, teachers can facilitate students' understanding of the material. Another alternative for teachers to facilitate students' conceptual mastery is by using representation (Perkins, 1998). Representation can be used as a measuring tool to identify students' conceptual understanding. Thus, the learning process aims to deepen students' understanding of complex mathematical concepts (Braseth, 2022).

Previous research (Rahmawati et al., 2017; You & Quinn, 2010) described representation as the basis for mathematic teaching and learning. In that teaching and learning process, different representational models are equally used and carry an important role in students' mathematic concept understanding (Duval, 2006; Presmeg, 1986). The representation transformation from one model to another is always necessary for a teaching or learning process. For example, students solve math problems using several representational models. Johnson (2018) expanded on Lesh's Model of representations into six catagories as illustrated in Figure 1. However, this research only focuses on the five main categories as proposed by Lesh's Model.

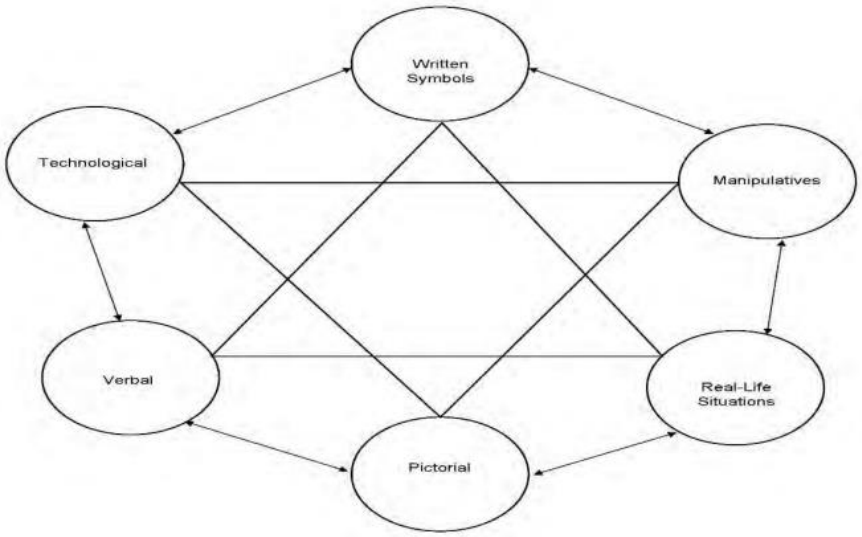

Figure 1. Johnson's Representational Models

From Figure 1, the explanation of each representation type of representation is explained in Table 2.

Table 2. Description of Johnson's Representational Model

Representation Type	Representation Indicators
Real Life	Knowledge-based on real-life events
Manipulatives	Objects that can be touched, moved, and often stacked like shards
Pictures	Pictures or diagrams
Verbals	Commonly used everyday language
Symbolic	Mathematical symbols are words or sentences
Technological	The use of technology that produces pictorial representation

The adoption of representation during learning can improve students' learning outcomes and understanding of mathematics (Anthony & Walshaw, 2009; Greeno et al., 1997; Santulli, 2009; Tripathi, 2008). This is because teachers' quality depends on their content and pedagogical knowledge in mathematics learning (Nurrahmawati et al., 2018). According to (Gagatsis & Shiakalli, 2004), the teachers' higher usage of representation in the learning process is proportional to students' mathematical understanding.

Previous research was conducted by Ngin (2018) with the aim of examining the multiple representation model from a commognitive perspective. The case study in this research focuses on an elementary school teacher who teaches the concept of percentages. The research results show that the use of a commognitive perspective increases teacher awareness when using multiple representations.

From the aforementioned studies, there has been no research examining multiple representations of fractions in each commognitive component during the teachers' material explanation. Previous research was limited to the use of types of representations, word problems, and manipulative nouns by teachers during teaching. Besides, previous studies mainly focus on students, not the teachers as student instructors. At the same time, the success of students relies heavily on the teacher's teaching procedure. Therefore, this study aims to investigate the teacher's use of multiple representations in teaching fractions based on the commognitive components perspective.

2. METHODS

This study used a qualitative approach with descriptive exploratory design to describe the teacher's commognitive in the learning process using multiple representations. Research was conducted by prioritizing activities during the learning process. The research participants were selected using a purposive sampling method. The research subjects were elementary school teachers who had educator certificates and more than 15 years of teaching experience. According to (Purnomo, 2021) older teachers present a lot of experience, judgment, and commitment. Further, that also on the level of teacher transferable skills. Thus, the research subjects were selected based on these criteria to achieve research objectives. The data were collected through recording when the teachers were teaching in the classroom. Then the recordings were analyzed to find the teacher's commognitive in the learning process using multiple representations.

The data used in this study were recorded learning videos and interview results. The recorded video was focused on the teacher's commognitive framework in the learning process using multiple representations. Besides, the additional critical data found during the data collection process were also collected and recorded. Furthermore, we also conducted an interview to confirm the obtained indicators of the teacher's commognitive framework and use of multiple representations.

The data were analyzed using the teacher's commognitive component indicators modeled by Sfard (2008) during the learning process, along with Johnson (2018) multi-representational model. Although Johnson has expanded the categories of representation to six, this research only focuses on five main categories (real life, manipulatives, pictures, verbal, and symbolic). The sixth representation, technological, is not used. We also conducted a data reduction stage by removing the irrelevant data for research purposes. Following the data reduction, we completed the data presentation and drew conclusions based on the reliability and validity measures used in the study. The process of uncovering the teacher's commognitive framework in the learning process using multiple representations was divided into stages, including the preparation stage, the data collection stage, and the data analysis stage.

3. FINDINGS AND DISCUSSION

In the teaching process presented, it was found that a commognitive component had emerged word use, visual mediator, routine, and narrative. This is shown when students respond to feeds from the teacher during learning. We provided the transcribed conversation between the teacher and students during the fraction learning.

Teacher : Pay attention all, sit pretty, pay attention to the teacher.
Student : Yes ma'am.
Teacher : Now the lesson is about fractions. Mathematics. There are mixed fractions and common fractions. Now the teacher gives an example. (while drawing bread on the blackboard).
Pay attention, children. Suppose you have bread, suppose this is your bread (while drawing). Your bread is one. Continue to share this one bread, if you have one younger sibling. This single bread continues to be divided into two. For you and your sister. For two, it's the same size. If dividing must be equal, it must be fair. If you divide, then divide by two. So, you get a half, your sister gets a half. How much is a half one?
(teacher draws a representation of ½ on the blackboard by shading one part and writing ½ on the blackboard)
Student : Half mom ..one and a half mom
Teacher : Keep going. If you have one bread (while drawing on the blackboard). Guess who had the bread? Try borrowing the teacher. Borrow Ms. Sifa's bread.
Ms. Sifa has bread. Is there only one bread? (the teacher shows the bread to the students).
This one bread is shared by your sister, you are half, your sister is half too. If you have two younger siblings, how? how much will you share?
Student : Three ma'am
Teacher : So, you have to divide by three. Come on, who can draw in fractions? Let's go forward, don't be afraid, later if something goes wrong it will be corrected.
Student : (silence)
Teacher : (the teacher repeats again). This is one loaf of bread (while draws again), if you divide it in half it will be half and half (while writes ½). Then if there is only one loaf (while drawing again), if your sister has three, it will automatically be divided by three (while writing 1/3 in the shaded part).
What are three called?
Student : denominator
Teacher : If one is called?
Student : Numerator
Teacher : Understand?
If divided by three, each part is 1/3. That is if you have three younger siblings.
What if there are four younger siblings and only one bread. (While shows bread)
How much is the bread for? Um…
Student : Four

Teacher : Four what? Four parts
(while drawing) this is for example a loaf of bread, dividing the loaf in half, now I will divide the other half in half (while drawing), how much will it be?
Student : quarter
Teacher : Clever.. a quarter (while writes ¼ on the picture of the fraction)
So each share is a quarter. So the parts are the same. Fair. Same.
Try it now pay attention. (while drawing fraction divided into four parts and shading two parts). So, how many fractions is this?
Student : Two by four
Teacher : Two-fourths correct (while writing 2/4)
So, do you remember what these two were called?
Student : Numerator
Teacher : What are the four called?
Student : Denominator
Teacher : What is two-fourths of this fraction called? Still remember? Fractional nor…
Student : Normal.
Teacher : Two quarters is equal to two divided by four. If four by two (while writing 4/2) is the same as four divided by two. What is four divided by two?
Student : Two
Teacher : Two times four how much? (while writing 2 x 4)
Student : Eight
Teacher : Twice two how much? (while writing 2 x 2)
Student : Four
Teacher : What is four divided by two? (while writing 4 : 2)
Student : Two

Description:

: Word use

: Visual mediator
: Routine
: Narrative

In learning fractions, the commognitive component used by teachers is the application of word use which describes the term fraction by utilizing visual media called visual mediators. The teacher explains the concept of fractions narratively using an object that is divided equally. Then the teacher also symbolizes this part using a fraction number consisting of a numerator and a denominator which is routinely applied in mathematics. It aims to convey the concept of fractions in a structured, clear and adequate manner, using various methods and visual representations to facilitate students' understanding effectively.

As described in the above transcript, the subject began by verbally explaining the contextual problem, namely dividing the bread among family members. Then, the teacher drew a rectangle to represent the bread and wrote the ($\frac{1}{2}$) fraction symbol. The rectangular image was divided into two equal parts, with one part being shaded. Then to ensure the students' understanding, the teacher took a real piece of bread and showed it directly to the students. Then, the teacher divided one piece of bread into two parts and each part is called one by two or half. The teacher repeated the explanation for one-third and one-fourth parts of the bread. After that, a detailed analysis of the commognitive components observed during the teaching process is presented in the following.

Table 3. Result of Teacher and Student Conversation Analysis

Commognitive Component	Activity	Used Representation
Word Use	The teacher uses words such as fractions, divided, divided equally, divided by two, half, half, one-half, half, quarter, quantifier, and denominator.	verbal: in the form of an oral explanation using the word fraction, divided, divided equally symbolic: $\frac{1}{2}$, $\frac{1}{4}$ image: the image of a rectangle divided in half by the same length real life: contextual problems when sharing bread with family members.
Visual Mediators	The teacher uses the media of pictures, bread, writing symbols	images: rectangular images divided into 2 equal sizes, rectangular images divided by 3 equal sizes, and rectangular images divided by 4 equivalent sizes. real life: bread is used as an authentic model for the problem related to fractional material symbolic: writing fractions into mathematical symbols manipulative: the teacher uses bread to illustrate the shape of fractions that are "assumed" to be the same size
Narrative	The teacher explains the definition of fractions using bread divided into two equal parts and bread divided into three equal parts.	real life: students are familiar with the distribution of bread in everyday life

		verbal: the teacher explains the process of dividing the bread to obtain equal parts for: $\frac{1}{2}$, $\frac{1}{3}$ manipulative: the teacher exemplifies the equal distribution of a loaf of bread
Routine	The teacher explains that 1 is divided into two equal parts, so the one part is called $\frac{1}{2}$. The teacher writes down the $\frac{4}{2}$, $\frac{2}{4}$, $2 \times 4, 2 \times 2, 4{:}2$ fractions. The teacher draws a representation of the $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ fractions.	symbolic: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{4}{2}$, $\frac{2}{4}$, 2×4, $2 \times 2, 4{:}2$, real life: the teacher explains the application of fraction material in everyday life verbal: the teacher explores students' understanding by telling stories manipulative: the teacher asks students to "imagine" the division of a piece of bread to obtain several equal parts

The commognitive component that begins with the selection of appropriate terms influences the way students match students' level of understanding, the process of association and conceptualization in students' minds can be more effective. Furthermore, the use of visual media and written symbols helps stimulate students' brains to visualize the concept of fractions. This helps the connection between the visual representation and the underlying mathematical concepts. In addition, a clear explanation of the concept of fractions with concrete examples and visualizing representations of fractions directly, provides an opportunity for students to build a strong mental model of the concept.

3.1 Word use

During the learning process, the teacher used several words, such as "fractions," "ordinary fractions," "divided," "equal," "divided by two, "divided by three, "divided by four," "half," "quantumber" and "denominator." The words "numerator," "denominator," "fraction," and "ordinary fraction" are in the category of mathematical terms, while "divided," "equal," "divided by two, "divided by three," "divided by four," "half," are included in the category of everyday words (Shuard & Rothery, 1984). Verbal representations aid the teacher in introducing the concept of fractions. Real-life representation is used by the teacher to start the lesson by telling the problems commonly encountered by students. Through verbal communication, the teacher can successfully explain the material, while the students can easily understand the material and solve a problem. Further, the teacher adopts image representation to exemplify the problem by showing an illustration in the form of a picture. Then, the teacher explains various kinds of words or terms that can be represented by symbolic representations, such as "half," which is written in ($\frac{1}{2}$). (Hiebert et al., 1997) explained the use of verbal representations could help students associate their everyday language and the mathematical language.

Strategic word use is critical in enhancing students' learning experiences with fractions. By carefully selecting and articulating mathematical vocabulary in a clear and accessible manner, teachers can dismantle linguistic barriers that often hinder students' comprehension of fraction concepts. For instance, simplifying mathematical terminology and providing contextual explanations enable students to grasp abstract fraction-related terms more readily. Additionally, actively incorporating fraction-specific vocabulary into daily instruction and providing ample opportunities for students to engage with and apply these terms in various contexts solidifies their understanding and fluency in communicating fraction concepts effectively.

3.2 *Visual Mediators*

In this learning process, the teachers use several media, such as blackboards, pictures of fractions, and manipulative representations of bread. The blackboard is used to write symbols and draw fraction representations. The picture fractions are represented in the form of a square image (illustrated in Figure 2a) that divided into two parts, with one part shaded showing ($\frac{1}{2}$) fraction (illustrated in Figure 2b), along with a square divided into three parts with one part shaded, showing the ($\frac{1}{3}$) fraction (illustrated in Figure 2c), and a square divided into four parts with one shaded part showing the ($\frac{1}{4}$) fraction(illustrated in Figure 2d).

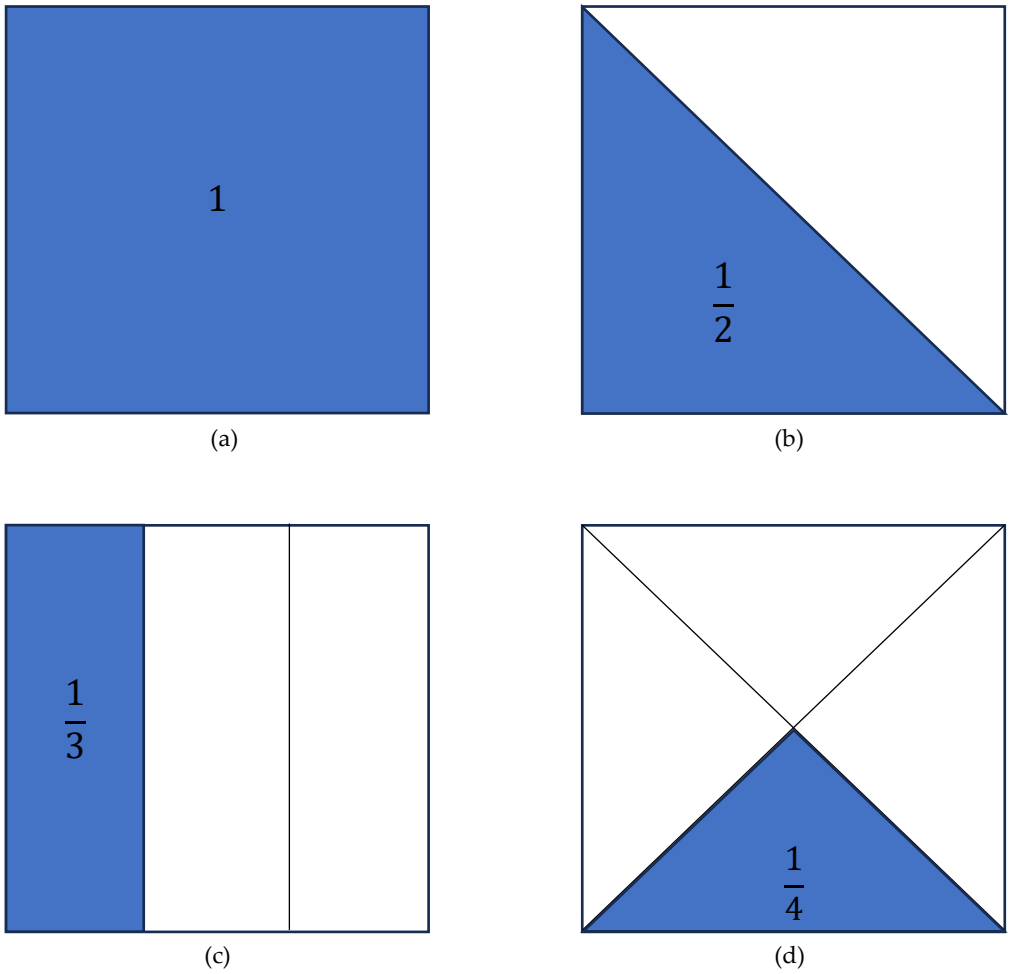

Figure 2 The Picture of Fractions

The visual mediators applied by the teacher during the fraction material learning process are illustrated in Figure 3.

Figure 3. Teacher Explains the Interpretation of Mathematics Problem

For the multi-representational form, the teacher used a square image divided into several parts and shaded according to fractional numbers. Besides, the teacher also uses another manipulative representation, such as the equal distribution of bread, as an application of the fractions concept to solve problems the authentic everyday life issues. Visual explanations aim to facilitate more meaningful problem construction (Prayitno et al., 2020). Teachers provide a variety of different visual representations to help students combine and improve their mathematical understanding (Rosita, 2019; Tomkelski et al., 2023). However, it is necessary to ensure that students avoid mistakes in implementing and interpreting the visuals with the precise interpretation of images (Nurrahmawati et al., 2021). Therefore, the teacher should also give guidance whenever students experience a discrepancy in the use of visual representations (Afriyani et al., 2019)(Afriyani et al., 2019) . Visual mediators are a practical commognitive component for teachers to help students comprehend symbolic representations of fractional. (Setyaningrum, 2017) shows that explanations using visual tools can help students understand the concept of fractions and improve their ability to solve fractions-related problems. Further, students can develop problem information through a representation using images and then turn it into a symbolic form, giving them the opportunity to solve problems independently. However, it should also be noted that some students perceive that the usage of symbols is complicated, so they prefer using simple mathematic calculations (Sa'adah et al., 2023).

Visual mediators enrich students' understanding of fractions by providing tangible representations of abstract mathematical concepts. Integrating visual aids such as diagrams, models, and manipulatives into instruction allows students to visually explore and manipulate fraction relationships, thereby fostering deeper conceptual understanding. For example, fraction squares enable students to visualize fractional parts and operations, making abstract concepts more concrete and accessible. Moreover, leveraging technology to incorporate interactive visualizations and virtual manipulatives enhances engagement and facilitates personalized learning experiences. By capitalizing on the multisensory nature of visual mediators, teachers can cater to diverse learning styles and empower students to develop a robust conceptual foundation in fractions.

3.3 Narrative

The teacher explains the introduction to fractional material using the narration of, "Suppose you have bread, suppose this is your bread (while drawing). Your bread is one. Then, you share this one loaf if you have a younger sibling." Through that narrative, the teacher uses examples of real-life contextual problems. The narrative given by the teacher to students contains the word "if" which shows a manipulative representation. In this process, the teacher gives examples or suppositions following the common issues faced by students. However, students often encounter obstacles in learning fraction material, such as in understanding the relationship between mathematical language and translating images into verbal form (Nurrahmawati et al., 2021). Consequently, specific efforts are needed to help students develop skills in understanding and translating during the learning process. Then, the teacher

emphasizes the explanation given "If it is divided in half, then it will be half and half (while writing ($\frac{1}{2}$))." This shows that the teacher uses verbal representations in the form of narratives to describe symbolic representations of fractions. Further, (Zayyadi et al., 2020) explained that content knowledge is necessary when writing proper mathematical notation in symbolic representations.

Incorporating narrative elements into fraction instruction offers a compelling approach to engage students and contextualize mathematical concepts within real-world scenarios. By weaving fraction problems into narratives or storytelling activities, teachers can provide students with meaningful contexts that resonate with their experiences and interests. For instance, presenting fraction challenges within storytelling prompts or problem-solving scenarios encourages students to apply mathematical concepts in authentic contexts, promoting more profound understanding and relevance. Furthermore, employing narrative techniques such as character development and plot progression captures students' attention and immerses them in learning. By leveraging the power of narrative, teachers can ignite students' curiosity and foster a genuine appreciation for the practical applications of fractions in everyday life.

3.4 Routines

The teacher applies real-life representations to explore students' understanding through contextual problems for fractional material as commognitive routines. These activities are carried out repeatedly and within a specified time according to the learning syllabus. In detail, the material delivered by the teacher is in accordance with basic competency 4.4 for the weather theme and the mathematics sub-matter, namely presenting fractions as part of a whole using concrete objects (Adelina et al., 2016). In this stage, the teacher explains that one is divided into two equal parts, so one part is called ($\frac{1}{2}$) using a verbal representation. Additionally, to increase students' understanding, the teacher usually uses manipulative representations by giving parables such as "For example, if a loaf of bread is divided in half, now I will divide the other half into two halves, how much will it be?" Then, the student answered, "a quarter". (Clark & Mayer, 2016) explains that student feedback on the teaching process will help enhance teaching effectiveness and improve the quality of learning. Besides, the teacher writes down the form of fractions based on the teacher's handbook that applies according to the curriculum, namely ($\frac{2}{4}$), ($\frac{4}{2}$), $2 \times 4, 2 \times 2, 4{:}\,2$ and describes the fractions ($\frac{1}{2}$), ($\frac{1}{3}$), ($\frac{1}{4}$) using symbolic representations. (Nurrahmawati et al., 2019) revealed that symbolic representations are often used in problem-solving questions in the form of stories related to everyday life.

Based on the teacher's commognitive component in explaining material using multiple representations, we observed two substantial discoveries. First, authentic real-life representation is used in each teacher's commognitive component. Real-life representations can help students understand problems, especially the ones commonly encountered in everyday life. The real-life representations are conveyed in the form of verbal, images, and also manipulatives, transforming these problems using mathematical language, namely symbols. (Rau & Matthews, 2017) explained that the teacher's use of multiple representations in fractional material facilitates their understanding of the concept of fractions more deeply and thoroughly. Students' ability to apply inter-representation correlates with the learning process carried out by the teacher (Nurrahmawati et al., 2021).

Second, the use of Symbolic representations has also been observed in each teacher's commognitive component because learning mathematics is always synonymous with symbols, according to the mathematics subject guidebook. Symbolic representation in the application of multiple representations in fractional material learning is very crucial because this concept involves complex and abstract rational numbers to develop a conceptual understanding (Anwar & Rahmawati, 2017). Visual representation of fractions in early grades is usually illustrative fractions as parts of a whole, like slices of a cake, this part-whole model can impact students' understanding of fractions as continuous to conceptualize fractions such as 9/8 (Hiebert & Tonnessen, 1978).

Establishing consistent routines and structured practices is instrumental in cultivating students' proficiency and confidence in working with fractions. Teachers provide students with predictable opportunities to engage with and reinforce fraction concepts by integrating regular routines for fraction exploration and practice into daily instruction. For example, implementing daily fraction review sessions or problem-solving routines instills a sense of continuity and progression in students' learning journeys. Additionally, scaffolding fraction instruction within a consistent framework helps students develop procedural fluency and problem-solving strategies over time. By embedding fraction routines into classroom activities, teachers create a supportive learning environment that encourages persistence, resilience, and continuous improvement in students' fraction proficiency.

In considering integrating word use, visual mediators, narrative techniques, and routines to enhance students' learning experiences with fractions, it is imperative to acknowledge potential limitations and avenues for future research. While these methods offer significant benefits in promoting comprehension and engagement, it is essential to recognize that they may only sometimes be used to handle all students' diverse needs and learning preferences. Additionally, further investigation is needed to explore the effectiveness of each of these approaches, for example, by considering different demographic groups, educational settings, and cultural contexts. For instance, research could explore the impact of word-use strategies on students with language barriers or the efficacy of narrative-based instruction in culturally diverse classrooms. Furthermore, follow-up research provides the potential to evaluate the durability and applicability of fraction skills obtained by regular practice. By critically examining the limitations of these methods and identifying gaps in current research, teachers and researchers can encourage a more balanced understanding of their utility and pave the way for future advancements in fraction instruction.

4. CONCLUSION

The results of this research state that teachers use multiple representations for each commognitive component according to learning needs (Zayyadi et al., 2020). By using commognitive components, teachers have many choices of learning models that can be implemented. Using varied representations can help students understand the concept of fractions better. So, teachers need to consider learning objectives and student characteristics when choosing a representation. Connecting representations with students' commognitive framework through appropriate strategies can increase the effectiveness of learning fractions. While these commognitive components have considerable benefits in terms of understanding and engagement, it is important to note that they may only be suitable for some students' unique requirements and learning preferences. Furthermore, further research is needed to investigate the efficacy of each of these techniques, such as taking into account diverse demographic groups, educational settings, and cultural contexts. This research also only involved one mathematics teacher in one learning class. The results of this study may only be generalizable to some teachers and classes. This research opens up opportunities for further research using a larger sample. Develop a comprehensive framework for representation-based teaching of fractions and study the effectiveness of using representations in learning fractions at different educational levels.

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