

Students' Mathematical Understanding Ability Through the M-Apos Approach on Derivative Materials

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ABSTRACT

This study aimed to improve students' mathematical understanding skills. The understanding ability is students' ability to understand the concept of the material, not just memorize the material. When students understand the concepts of differential calculus, students can quickly solve problems in differential calculus. The students' understanding gained in the differential calculus material will be the basis for understanding the integral and advanced calculus material. However, the current conditions where the Coronavirus that has hit the world, including Indonesia, have resulted in students having to study from home with a learning process that is carried out online. Students cannot access books in the library and limited credit to access teaching materials from the internet because the learning materials owned by students are limited. It makes it difficult for students to learn, so students' mathematical understanding abilities are low. The M-APOS learning approach was used in this study to answer this problem. The study results found that learning with this approach has succeeded in increasing students' mathematical understanding abilities on derivative material. This research is an experimental study with a pretest-posttest control group research design.

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1. INTRODUCTION

A derivative is one of the topics studied in the differential calculus course. Derivative material has been studied starting from high school education. A derivative is one of the basic materials that students must master. Apart from being included in the differential calculus course, this material is also studied in other courses such as integral calculus, advanced calculus, differential equations, and other courses. In studying calculus, derivative material plays an important role. However, many

students still have difficulty solving problems or questions related to derivative material because their basic skills regarding derivative material are still deficient.

Students as prospective teachers are required to have the ability to understand differential calculus courses because differential calculus is taught in high school. However, in contrast to the importance of understanding ability, the facts in the field show that students' understanding ability on the topic of differential calculus is still low (Mendezabal & Tindowen, 2018; de Almeida et al., 2021). The Covid 19 that hit Indonesia required students to study from home. Students cannot access books in the library, and limited credits to access the material provided cause students difficulty in obtaining materials about the material being taught. The lack of teaching materials also makes learning difficult, so students' understanding abilities are low (Spitzer & Musslick, 2021).

Several factors cause students' low mathematical understanding ability, namely 1) inappropriate learning, 2) learning that only asks students to solve routine problems, or is rote tending to make students passive in learning (Mabena et al., 2021). In addition, there are limited learning resources such as textbooks on differential calculus, and the learning approach used is still conventional. Furthermore, Ismail et al. (2015) state that the ability to understand mathematics is (1) Re-expressing the understanding of mathematical concepts in their language, (2) Identifying mathematical concepts in problems and explaining the relationship between concepts to solve problems, (3) Comparing and distinguishing mathematical concepts, and (4) Changing one form of representation to another form.

Mathematics understanding is essential for students because students with mathematical understanding will be able to connect mathematical theories or formulas with problems in the world of work (Lin et al., 2017). Students who understand mathematics define mathematical principles in a more simplified form (Ukobizaba et al., 2021). Next, they connect what is happening in the field with mathematical principles. If the problem is new, students who have an understanding will be able to know the connection between new concepts and old concepts (Yuanita et al., 2018). Therefore, the M-APOS approach is used to improve students' understanding abilities.

The M-APOS or Modified-APOS approach stands for action, process, object, and schema. The M-APOS approach is a modification of the ACE (activities, class discussion, exercises) approach based on the APOS (action, process, object, schema) theory. This approach can improve the ability of mathematical creativity and strategic competence. The M-APOS approach encourages students to learn actively and meaningfully through four stages: action, process, object, and scheme (Saefudin & Kintoko, 2018). The M-APOS approach includes giving recitation assignments as an activity so that students form their knowledge (Borji et al., 2018). Furthermore, the M-APOS approach makes students have discussions with other students, resulting in an understanding of the material being studied (Maharaj, 2013). Students who study with the M-APOS approach will experience meaningful learning where students are aware of and know the reasons for using rules, principles, and formulas (Arnawa et al., 2021). Another study by Lestari (2014) suggests that learning using M-APOS can improve the ability to understand concepts and students' motivation to learn calculus II. From the research results that have been found, what distinguishes this research is the indicators of understanding ability used, materials, samples, variables, and data analysis used.

Based on the description above, the problem in this research is whether the M-APOS learning approach can improve students' understanding of derivative material. Meanwhile, the goal was to improve students' mathematical understanding skills through the M-APOS approach to derivative material.

2. METHODS

This type of research is quasi-experimental. In this quasi-experimental study, the subjects were not grouped randomly. However, the researcher accepted the subject's condition as modest (Bärnighausen et al., 2017). In this study, two classes were used: the experimental and control classes. The initial stage of this research was to determine the research sample. Then two classes were taken

randomly, namely one class as the experimental class and the other for the control class. This treatment was given to see its effect on the measured aspect: the student's mathematical understanding ability. The research design used in this study was as follows. The form of the design used in this study is shown in Table 1.

Table 1. Pretest-posttest research design

Group	Pre-test	Action	Post-test
Experiment Class	O ₁	X ₁	O ₂
Control Class	O ₃	X ₂	O ₄

Where:

X₁ : Treatment using the M-APOS approach

X₂ : Treatment using the conventional approach

O₁ : The results of the class pretest using the M-APOS approach

O₂ : The results of the class posttest using the M-APOS approach

O₃ : The results of the class pretest using the conventional approach

O₄ : The results of the class post-test using the conventional approach

The population of this study was all first-semester students, totaling 66 people. Therefore, the sample in this study was a saturated sample or a population sample of 66 students. The sample was a Mathematics Education Study Program student, Department of Mathematics and Natural Sciences, FKIP, Pattimura University. The selection of Mathematics education undergraduate students as research samples was based on the consideration that the level of cognitive development of students was at the formal operational stage, so it was deemed appropriate to use the M-APOS approach. In addition, first-semester students were still in the stage of adolescence. At this time, students were in the process of self-discovery/identity and self-confidence formation. Therefore, some materials were predicted to be suitable for use in learning with the M-APOS Approach so that students' mathematical understanding abilities could develop.

The variables of this study consisted of three variables: (1) the independent variables included the M-APOS learning approach; (2) dependent variables, including students' mathematical understanding ability; The instrument used to measure students' mathematical understanding ability of derivative material was a test instrument. The test given was in the form of an essay question consisting of 4 questions. Indicators of mathematical understanding ability used were (1) Expressing the understanding of mathematical concepts in their language, (2) Identifying mathematical concepts in problems, and explaining relationships between concepts to solve problems, (3) Comparing and differentiating mathematical concepts, and (4) Changing one form of representation to another.

The data collection technique in this study was a test technique. The intended test was a mathematical understanding ability test that was carried out after the learning took place. This test was given to improve the understanding ability of students of the Mathematics Education Study Program after being given learning using the M-APOS approach.

There were two types of data in this study, namely quantitative data and qualitative data. Quantitative data was obtained through analysis of student answers on the student's understanding ability test, then grouped based on the learning approach used, namely the M-APOS learning approach and the conventional learning approach. Qualitative data were obtained through observations of the activities of lecturers and students in implementing learning and data from interviews with students. This data was analyzed descriptively to support the completeness of quantitative data in answering research questions.

The quantitative data processing was carried out through several stages, namely:

The first stage was performing a descriptive analysis of the data and calculating the value of Pretest, Posttest, Gain, and Normalized Gain. With this stage, it can be seen how big the achievement, the increase in students' understanding ability in the class that uses the M-APOS learning approach

and the class that uses the conventional learning approach, then the normalized gain was calculated $\langle g \rangle$ (normalized gain score)

The calculation of the normalized Gain score was carried out because this study not only looked at the increase in students but also saw the quality of the increase. In addition, the normalized Gain score calculation was carried out to eliminate the student guess factor and the effect of the highest score to avoid biased conclusions (Hake, 1998). The $\langle g \rangle$ value ranges from 0 to 1, then the $\langle g \rangle$ value was processed, and the processing was adjusted to the problems and hypotheses proposed. In the second stage, statistical prerequisite tests were carried out as the basis for hypothesis testing, namely the normality test for the distribution of the sample data and the homogeneity test of variance for parts or the whole group. The third stage was determining the increase in students' mathematical understanding abilities between the experimental and control classes, determining whether or not there was an interaction between the independent variable and the control variable on the dependent variable following the hypothesis. The average difference test was used using the assistance of the SPSS 22.0 program for Windows to test the difference. In addition to quantitative analysis, the researcher conducted a qualitative analysis of the answers to each question, observation data, and student response data. It aimed to examine students' understanding abilities further and to find out whether the implementation of learning is following the stipulated learning provisions on both learning models.

3. FINDINGS AND DISCUSSION

According to the problems raised and the objectives of this study, data analysis was carried out by testing the differences in the improvement of students' mathematical understanding abilities who received learning with the M-APOS approach and the conventional approach used the Average Difference Test.

Data Analysis of Improving Mathematical Comprehension Ability

1) Description of the Normality Test Data for Mathematical Comprehension Ability

The results of the Normality of the N-Gain Data on Students' Mathematical Understanding Ability in the Experiment Class and Control Class are shown in Table 2.

Table 2. Descriptive statistics mathematical comprehension ability score

	Control Class (Conventional)					Experiment Class (M-APOS)				
	N	\bar{x}	s	x_{min}	x_{maks}	n	\bar{x}	s	x_{min}	x_{max}
Pre-test	32	55,12	8,87	36	70	34	51,29	8,24	3	20
Post-test	32	80,56	5,65	70	91	34	82,88	6,17	72	98
N-gain	32	0,56	0,09	0,35	0,76	34	0,65	0,11	0,43	0,95

Ideal Maximum Score: 100

From the data in Table 1, it can be seen that the scores of students' mathematical understanding abilities on derivative material for the average pre-test, post-test and N-Gain in classes that use the M-APOS learning approach and the conventional learning approach each look different.

2) Classification of Students Based on N-Gain Mathematical Comprehension Ability

The results of the improvement category for the M-APOS class and the Conventional class are shown in Table 3.

Table 3. Classification of students based on n-gain ability of mathematical understanding

Class	Upgrade Category	N	\bar{x}	Percentage (%)
Conventional	High	4	0,72	12,50
	Medium	28	0,54	87,50
	Low	-	-	-
M-APOS	High	10	0,77	29,41
	Medium	24	0,60	70,59
	Low	-	-	-

The data in Table 2 shows that the classification of students based on the N-Gain of mathematical understanding ability for the experimental class (M-APOS) and control class (Conventional) has different averages and percentages in each category. From the table above, it can also be seen that for classes that use the M-APOS learning approach and classes that use a learning approach, both do not have low categories.

3) Normality Test Results Pretest, Posttest and N-Gain Mathematical Comprehension Ability

The results of the normality test of the Pre-test, Post-test, and N-Gain data for students in the Experimental Class (M-APOS) and Control Class (Conventional) are shown in Table 4.

Table 4. The normality test results of pretest, posttest and n-gain of mathematical understanding ability

	Classes	Shapiro-Wilk Uji test		
		Statistic	df	Sig.
Pre-test	Conventional	0,968	32	0,446
	M-APOS	0,979	34	0,742
Post-test	Conventional	0,971	32	0,516
	M-APOS	0,974	34	0,580
N-Gain	Conventional	0,964	32	0,352
	M-APOS	0,976	34	0,640

Based on table 3, it can be seen that the results of the normality test of the pre-test, post-test and N-Gain data for the two learning groups using the Shapiro-Wilk test for students' mathematical understanding abilities on derivative material indicate that the data are normally distributed.

4) Results of Homogeneity of Pretest, Posttest and N-Gain of Mathematical Understanding Ability

The results of the homogeneity test of the Pretest, Post-test and N-Gain data for students in the Experimental Class (M-APOS) and Control Class (Conventional) are shown in Table 5.

Table 5. The homogeneity test results of pretest, posttest and n-gain of mathematical comprehension ability

	Classes	Levene test		
		Statistic	df2	Sig.
Pre-test	Conventional	0,241	64	0,625
	M-APOS			
Post-test	Conventional	0,58	64	0,810
	M-APOS			
N-Gain	Conventional	0,155	64	0,695

From the data in Table 4, it can be seen that the results of the homogeneity test of the pre-test, post-test, and N-Gain data of the two learning groups for students' mathematical understanding abilities on derivative material indicate that these data are homogeneous.

5) The Mean Differences Test Results of Pre-test, Post-test and N-Gain of Mathematical Comprehension Ability

From the results of the normality and homogeneity tests for the pre-test, post-test, and N-Gain data, shows that the data for both groups are normally distributed and homogeneous so that a mean difference test can be performed for the pre-test, post-test and N-gain data of mathematical understanding ability as shown in Table 6.

Table 6. the mean difference test results of pre-test, post-test and n-gain of mathematical comprehension ability

	Class	t-test	Asymp.Sig. (2-tailed)	Asymp.Sig. (1-tailed)
Pre-test	Conventional	1,818	0,74	0,37
	M-APOS			
Pos-test	Conventional	-1,589	0,117	0,0585
	M-APOS			
N-Gain	Conventional	-3,431	0,001	0,0005

Data Table 6 shows the value of Sig. (2-tailed) for Pre-test data is 0.74. It is greater than $= 0.05$, so it can be concluded that there is no difference in students' mathematical understanding ability between the experimental class (M-APOS) and control class (conventional). Then, the value Sig. (2-tailed) for Post-test data is 0.117, which is greater than $= 0.05$. Therefore, it can be concluded that there is no difference in students' mathematical understanding ability between the experimental class (M-APOS) and control class (Conventional), while the value of Sig. (2-tailed) for the N-Gain data is 0.001, which is smaller than $= 0.05$. It shows differences in students' mathematical understanding abilities on derivative material between classes using the M-APOS learning approach and classes using the conventional learning approach.

Based on the results of research using the M-APOS learning approach and conventional learning approaches on derivative material, it can be seen that there is no difference in increasing mathematical understanding abilities on pre-test scores between students taught with the M-APOS learning approach and conventional learning approaches. Pre-test scores were obtained from tests given to students before learning activities were carried out. It was done to see students' initial ability about the material to be studied. The test was carried out online. It is due to the current situation of the COVID-19 pandemic that has hit the world. Therefore, all activities were carried out online. The same thing also happened to the post-test scores, which showed no significant difference in improving students' mathematical understanding skills between the experimental group (M-APOS learning approach) and the control group (conventional learning approach). Post-test scores were obtained from tests given to students after the learning activities. In this study, learning is carried out online. It happens because of the COVID-19 pandemic hitting our world today, so all learning activities from PAUD to Higher Education levels cannot be carried out face-to-face. Meanwhile, the N-Gain value shows a significant difference in increasing students' mathematical understanding skills between the experimental group (M-APOS learning approach) and the control group (conventional learning approach).

From these results, it can be seen that in the teaching and learning process, lecturers must pay attention to matters related to learning, including the learning model used, learning approach, learning strategy or learning method including the classroom atmosphere when learning takes place

because this can also affect students' thinking. It is also because the M-APOS learning approach makes students more active in the learning process. Students are allowed to construct their knowledge by working on Student Worksheets (LKM) and can express their opinions so that the learning outcomes are more meaningful and students can understand what they are learning.

In line with the above, understanding is one of the abilities that is the goal of learning mathematics in the curriculum, namely understanding ideas with symbols, tables, diagrams, or other media to clarify situations or problems (Laurens et al., 2018). The ability to understand mathematics is also one aspect of the standard of the mathematics learning process according to the recommendations of the NCTM (2000). The indicators of understanding ability recommended by NCTM (2000) include 1) defining concepts verbally and in writing, 2) identifying and making examples and non-examples, 3) using models, diagrams, and symbols to represent a concept, 4) changing a form of representation to other forms, 5) recognizing the various meanings and interpretations of concepts 6) identifying the properties of a concept and recognizing the conditions that determine a concept, 7) comparing and differentiating concepts. Understanding as part of the standard mathematics learning process helps build concepts and strengthen students' ideas.

Students' mathematical understanding ability in learning is important to achieve learning objectives. Students who understand the material or mathematical concepts will be seen from how the student solves the given problem (Fouze & Amit, 2018). Understanding mathematics can be said to be the foundation for developing mathematics learning. The ability to understand mathematics is a basic aspect that influences the growth of other mathematical abilities (Schoenfeld, 2016). Therefore, students cannot develop other abilities if they do not understand the material being studied. Understanding new concepts are closely related to students' understanding of previous concepts. Therefore, when students do not understand previous concepts, students will find it difficult to explain or do a proof in mathematics (Davies et al., 2021).

Achieving an understanding ability in mathematics is not an easy thing because the ability to understand material or concepts in mathematics from each individual or student is different. It must also be supported by a good classroom situation so that the learning process can run smoothly.

4. CONCLUSION

Based on the description of the results and discussion, it can be concluded that (1) there is no difference in increasing mathematical understanding ability for pre-test scores between students, (2) there is no difference in increasing mathematical understanding ability for post-test scores between students who are taught by using M-APOS learning approach and conventional learning approach, (3) there are differences in the increase in mathematical understanding abilities for the N-Gain value between students who are taught with the M-APOS learning approach and conventional learning approach.

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